

Summary

Majorana fermions have never been observed as fundamental particles. However, they appear in various systems as low energy excitations - quasiparticles. Moreover, ordinary Standard Model neutrinos as well as a dark matter particle may appear to be Majorana fermions. This thesis studies different aspects and properties of the Majorana fermions in free and interacting systems.

Lowest energy excitations in topological superconductors have Majorana nature (called Majorana zero modes or MZMs). These modes are not fermions in a normal sense, but when two of them are coupled such that their wavefunctions overlap (fused) - they form a fermion. Such MZMs are spatially separated, pinned to have exactly zero energy, are free of decoherence and have a non-abelian anyonic statistics under exchange. One can see signatures of MZMs and their anyonic statistics through fusion. If one prepares four MZMs (denoted γ_i) in the state where γ_1, γ_2 and γ_3, γ_4 form states with definite fermion parity, the outcome for the parity of the state formed from γ_2, γ_3 will be non-deterministic, namely $\langle i\gamma_2\gamma_3 \rangle = 0$. This happens for an ideal system where the MZMs are well separated from the continuum. However, as was shown in chapter 2, in a real system with some degree of disorder this is no longer a distinctive feature of non-abelian MZMs. The underlying reason for this is an unfortunate coincidence that the systems that host topologically protected MZMs are also prone to accumulate other parasitic states near zero energy that can mimic the true MZMs in the observables.

Nevertheless, as is shown in chapter 3, one can overcome this obstacle by looking at the dynamical signatures of the ground-state degeneracy. In particular, one needs to compare the time-dependent evolution in the parameter space of coupling constants via two alternative pathways. The topological ground-state degeneracy of Majorana zero-modes causes a breakdown of adiabaticity that can be measured as a pathway-dependent fermion parity. The correlation between two pathways for the accidental degeneracies of the Andreev levels is distinct from what would follow from the Majorana fusion rule.

An opposite case of strongly interacting Majorana fermions becomes very interesting if the number of fermions N is big, interactions are all-to-all and randomly distributed. Such a model is called Sachdev-Ye-Kitaev model (SYK) and it comprises several peculiar properties: it possesses an exact solution at strong coupling lacking quasiparticles, it has an emergent conformal symmetry in the infrared and it saturates the upper bound on quantum chaos. In this thesis we study how these properties manifest themselves in observables. In particular, in chapter 4 we find that if one couples the SYK system to a superconductor, upon increasing the coupling strength up to the critical value, the pairing gap Δ behaves as $\eta\hbar/t_P$ at low temperatures, where $\eta \sim 1$. The lower critical temperature emerges with a further increase of the coupling strength so that the finite Δ domain is settled between the two critical temperatures. This does not happen if one, instead, couples the superconductor to disordered but non-interacting fermions. In that case, upon increasing the coupling the superconductivity just dies out.

In chapter 5 we studied SYK system at initial temperature T coupled by a quench to a large fermionic reservoir kept at zero temperature. In such a system, a tunneling current appears and the dynamics of the discharging process of the SYK quantum dot reveals a distinctive characteristic of the non-Fermi liquid state. In particular, the current's half-life scales linearly in T at low temperatures, while for the Fermi liquid it scales as T^2 .

The last part of this thesis which is chapters 6,7 and 8 is devoted to an experiment that aims at the detection of relic neutrinos. We show that in case of solid state base experimental architecture, namely when the β -decayers are attached to some kind of substrate (which is the only viable possibility so far) there is a fundamental intrinsic limitation on its energy resolution. It comes from a simple Heisenberg's uncertainty principle: when we restrict the β -emitter to a finite volume in space, it acquires uncertainty in its momentum which smears the spectrum. We also show that the only way to mitigate this effect is to use a heavier β -emitter, more detailed study shows that the only viable candidate is ^{171}Tm . However, this does not solve all the problems since solid state materials host a whole zoo of elementary excitations that affect the intrinsic uncertainty of the detector through a range of mechanisms. The investigation of all these mechanisms and finding ways of mitigation requires close collaboration between high-level experts in both theoretical and experimental solid state physics and may lead to further modifications of the experimental architecture.