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Airport Capacity Modelling for Dynamic Separation Distance Schemes

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Summary

With the steady increase in air traffic, nowadays most of the major hub airports experience severe traffic congestion during certain portions of the day. The main limitation at the moment is found to be the amount of landing aircraft an airport can accommodate during a period of time. A specific runway capacity constraint is the required minimum wake turbulence separation distance between consecutive landing aircraft. Currently research is being done to determine safe reduced wake vortex separation constraints that dynamically alter with the occurring local weather type.

This document describes the work undertaken in the final part of the author's studies at the mathematics department of the Universiteit Leiden, carried out at the Nationaal Lucht- en Ruimtevaartlaboratorium (NLR). The objective was to develop an airport runway capacity model capable of investigating the direct benefits in the application of weather dependent reduced distance separation requirements. The effort resulted in a further development and implementation of existing analytical runway capacity models.

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List of Abbreviations

ATC	Air Traffic Control
ATM	Air Traffic Management
FAA	Federal Aviation Administration
FANOMOS	Flight track and Noise Monitoring System
ICAO	International Civil Aviation Organisation
IFR	Instrument Flight Rules
IMC	Instrument Meteorological Conditions
LMI	Logistics Management Institute
LVNL	Luchtverkeersleiding Nederland
NASA	National Aeronautics ans Space Administration
NLR	Nationaal Lucht- en Ruimtevaartlaboratorium
NM	Nautical Mile
ROT	Runway Occupancy Time
VFR	Visual Flight Rules
VMC	Visual Meteorological Conditions

1 Introduction

1.1 Context

With the steady increase in air traffic, nowadays most of the major hub airports experience severe traffic congestion during certain portions of the day. The main limitation at the moment is found to be the amount of landing aircraft an airport can accommodate during a period of time. Therefore not surprisingly, large delays occur in particular during an airfield's arrival peaks as the landing facilities become saturated. Research is undertaken to gain understanding in the causes of restrictions and to recognise methods to relieve them. The ultimate goal of such analyses is to attain an improvement of the present situation at airports and to prevent an increasing occurance of air traffic congestion in the future, henceforth increasing the airport runway capacity.

1.2 Scope

Most of this paper will concern airport runway capacity enhancements during Instrument Meteorological Conditions. Under these circumstances, international safety regulations prescribe minimum wake turbulence separation distances between consecutive landing aircraft. These separation standards evolved over time to prevent wake encounters in weather conditions conductive to long-lived wakes. During weather circumstances that lead to a rapid wake decay or a vortex motion away from the flight path of a following aircraft, the distance separation constraints can be considered overly conservative and abundantly limit the runway usage capacity. Since both wake decay and motion are highly influenced by ambient meteorological conditions, research is being done to determine safe reduced wake vortex separation constraints that may dynamically alter with the occurring local weather type. This could provide a clear opportunity to enhance airport runway capacity during favourable weather conditions. Beside the longitudinal aircraft vortex spacing requirements there are several other aspects that can affect an airport's runway capacity. For instance, elements as the pilot and controller performances on the tactical side whereas strategic elements include the traffic mix or the arrival schedule, the airfield infrastructure and the runway mode of operations. In addition, the airport runway capacity can be limited by government imposed environmental constraints.

1.3 Objectives

To predict actual benefits of proposed improvements, one needs to acquire models that adequately describe the situation and that permit a qualitative study of the essential features involved. The objective of this paper is to propose a mathematical model to investigate airport capacity enhancements as a consequence of the introduction of weather dependent reduced distance separation requirements. Instead of providing just a mere summary of previous results, the proposed model will attempt to offer a new approach to the airport runway capacity modelling problem.

1.4 Outline of this document

Section 2 briefly discusses some factors of influence on the airport runway capacity. In Section 3 an overview of currently existing analytical runway capacity models is provided and a review is given of research on non-stationary queuing model approximations to the delay experienced by congested aircraft. Section 4 states some properties an airport runway capacity model should preferably take account of. In Sections 5 and 6 this paper's capacity model is proposed. Section 5 discusses the model's internal logic and in Section 6 the model parameters are derived. In Section 7 some model results are provided. Section 8 discusses some possible future model extensions. Section 9 states conclusions and recommendations. A brief description of developed software tools is contained in Appendix A.

2 Influences on runway capacity

To provide some general insight into the aircraft arrival process and corresponding air traffic procedures, and to gain some further understanding in some of the potential capacity influencing elements mentioned in the introduction and their possible effects on an airfield's runway capacity, a brief review is provided in the next sections.

2.1 Airfield surroundings and runway occupancy

Aspects of the airfield surroundings, for instance, contain the airfield infrastructure, the runway mode of operations and environmental constraints.

2.1.1 Airfield infrastructure

The airfield infrastructure consists of the configuration of the runways, the location of the runway exits, the taxiways and aprons. As safety regulations limit the use of a runway to one single aircraft at a time, the time needed to perform operations on a runway, the Runway Occupancy Time (ROT) may pose capacity limitations. This runway occupancy time can be influenced by numerous factors. For an arriving aircraft, pilot performance aspects include the years of experience on the aircraft type, the familiarity with the layout of the particular destination airport and the awareness of the actual need to minimise the runway occupancy time. A landing aircraft occupies a runway once it has crossed the runway threshold. Although the touch down is generally aimed at a designated point, usually located some 300 meters beyond the runway threshold, there are various reasons for an aircraft to gradually float further along the runway. On the ground the landing weight and speed, the preliminary selected auto brake setting and the optional use of reverse engine thrust are factors determining an aircraft's ability to decelerate to taxi speed. Once taxiing, the aircraft's turning capability in particular is the main limiting factor. In addition, the actual selected runway exit to turn off may in practice not be the most favourable exit in terms of runway occupancy, as passenger comfort and aircraft brake wear also tend to form important considerations.

2.1.2 Runway mode of operation

The runway mode of operations describes the sort of operations that are actively allocated to a particular runway. These operations can be arrivals only, departures only and mixed operations for generally one of the directions the runway can potentially be used. As an immediate consequence of the airfield infrastructure, it is possible for a runway to have a different maximum landing capacity in terms of runway occupancy for the two directions of usage, for instance, due to a different location, visibility and type of exits.

2.1.3 Environmental constraints

The major environmental constraint to most airports is provided by noise abating restrictions as imposed by local federal authorities. Such noise monitoring can result in direct restrictions like runways used only during certain periods of the day or indirect restrictions on the total amount, sort and direction of operations performed on a particular runway over a longer period of time. Such noise constraints can thus have a critical impact on the availability of runway configurations and their functional mode of operations.

2.2 Weather influences and separation criteria

Clearly, rare meteorological effects such as heavy thunder or snow storms can have a severe impairing effect on airport runway capacity. More ordinary weather induced components such as wind speed and direction can influence the direct availability of the several runway configurations and their modes of operations. Another rather common effect is provided by the fact that, whether a runway is wet or dry can induce differences in runway occupancy times, since generally a longer brake way is needed in the former case, as then the taxi speed, at which an aircraft can safely perform the exit turn off, significantly drops.

2.2.1 Meteorological conditions

In addition to Instrument Meteorological Conditions (IMC) as mentioned before in the introduction, also Visual Meteorological Conditions (VMC) can occur and for both of these weather classes certain safety regulations apply. Within these regulations also different categories or gradations of both the VMC and the IMC weather class are distinguished.

Visual meteorological conditions

In the most favourable VMC class, the cloud ceiling and visibility level are beyond certain critical limits. Under these circumstances the Visual Flight Rules (VFR) may apply and the responsibility of maintaining enough separation between the landing aircraft is then in hands of the pilots. By using their knowledge of the current local weather, the leading aircraft type and its observed flight path they can effectively self separate from wake turbulence encounters.

Instrument meteorological conditions

As weather conditions deteriorate, a loss of visual approach capability decreases runway capacity due to numerous reasons. These include the unavailability of certain runway configurations, such as closely spaced parallel runways where visibility is a dominant factor, which generally results in a reduced number of active runways, and the application of longitudinal wake vortex separations under the responsibility of the Air Traffic Control (ATC). These wake vortex separations evolved over time to prevent aircraft from dangerous turbulence encounters in the near vicinity

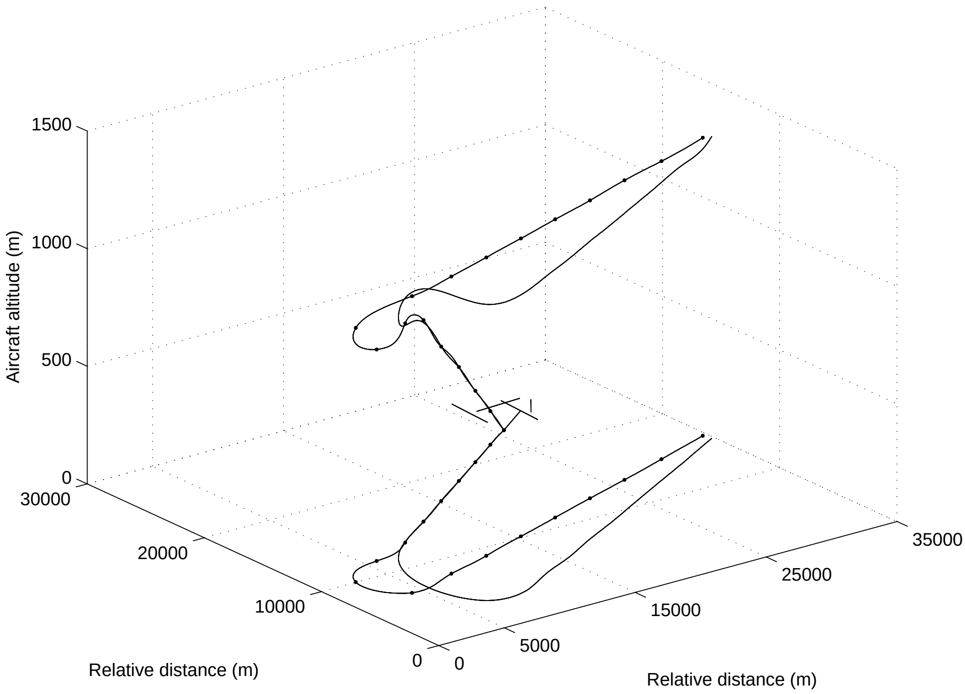

Fig. 1 Flight paths of consecutive arrivals on Schiphol Airport runway 06 as recorded by a Flight track and Noise Monitoring System (FANOMOS) in the early morning of April 3^{rd} 1999. The aircraft follow a $3°$ approach glide slope. For clarity, path projections on the ground surface are included.

of the ground. During IMC the Instrument Flight Rules (IFR) prescribe that landing aircraft are guided towards the runway, for instance by means of the Instrument Landing System (ILS). As a consequence, deviations in the followed approach path towards the runway tend to be relatively small in the final phase of flight. The underlying thought with applying longitudinal separations, is that, when provided with a minimum spacing, the generated wake has time to diminish to an acceptable operational strength for the following aircraft. To obtain a measure of the differences in consecutive flight paths, especially during final approach, one is referred to Figure 1. Near the runway the corridor through which the aircraft pass is rather narrow.

2.2.2 Wake turbulence categorisation

In view of wake vortex encounters by landing aircraft, the combination of any aircraft landing behind a larger aircraft on the same, a closely spaced parallel or even a crossing runway are known for their enhanced turbulence related risk. The International Civil Aviation Organisation (ICAO, Ref. [12]) currently provides the following wake turbulence categorisation.

LIGHT:	for an aircraft type with a maximum certified take-off weight of 7.000 kg or less.
MEDIUM:	for an aircraft type with a maximum certified take-off weight less than 136.000 kg but more than 7.000 kg.
HEAVY:	for an aircraft type with a maximum certified take-off weight of 136.000 kg or more whether or not they are operating at this weight during a particular phase of flight.

Since the potential severeness of the occurring wake turbulence is directly related to the amount of lift needed by the generating aircraft to maintain airborne, this classification is made according to the aicraft maximum certified take-off weight.

2.2.3 Wake turbulence separation minima

Given the classification provided in the previous section, ICAO regulations (Ref. [11]) provide the longitudinal separation minima for successive arriving aircraft pairs. These separation distances are listed in Table 1.

	Following aircraft class		
Leading aircraft class	HEAVY	MEDIUM	LIGHT
HEAVY	4.0 NM (7.4 km)	5.0 NM (9.3 km)	6.0 NM (11.1 km)
MEDIUM	3.0 NM (5.6 km)	3.0 NM (5.6 km)	5.0 NM (9.3 km)
LIGHT	3.0 NM (5.6 km)	3.0 NM (5.6 km)	3.0 NM (5.6 km)

Table 1 ICAO separation minima for arriving aircraft.

The wake turbulence separation minima are intended to reduce the potential wake turbulence related hazards.

2.3 Controller performance and traffic mix

The controller capability can be a clear bottleneck to the runway capacity if the workload of the tower controllers given saturation circumstances cannot be safely handled. In this paper no reference is being made to the psychological aspects of traffic controlling and it is assumed that the ATC is fully equipped and capable of handling congested traffic. Even under this assumption there are still some crucial factors that cannot be fully controlled.

2.3.1 Traffic mix

The traffic mix or arrival schedule may vary with the time of the day, day of the week and season of the year. On operating an airfield runway, the allowed separation minima on both taking off and landing are directly determined by the wake turbulence weight categories for the consecutive aircraft. While the ATC has some traffic managing abilities to optimise the aircraft sequencing, the traffic mix still highly influences the probabilities of occurrence for the respective different alternations of aircraft within a landing stream.

2.3.2 Controller precision

Within an aircraft landing sequence, there is a limit to the precision at which the ATC can place an individual aircraft at a certain position. An important fact here is that no exact information on the actual location of the aircraft is available to the ATC, as such information is only refreshed once every few seconds depending on the radar frequency. Given the aircraft's speed, this uncertainty may result in a region of a few hundreds of meters. Another shortcoming in the amount of information at the ATC's disposal is found in the aircraft speed itself. Up to a certain moment the ATC may issue speed warrants. Given such speed prescriptions, there is an uncertainty to which extend these speed regulations can be accurately carried out, for example, in gusty wind conditions. As the aircraft draws near the runway threshold, typically from a distance of about 4NM but even up to a distance of 9NM off the runway threshold depending on the ATC level of speed control, it is up to the pilot to determine a safe final approach speed. The actual final approach speed is closely related to the aircraft stall speed, which depends on the aircraft's actual landing weight. Given these circumstances, it is the ATC's responsibility to control the air traffic, taking account of safety procedures. It is needless to mention that rare traffic with significantly different characteristics poses a disproportionate amount of extra stress on the managing process and that, in the absence of accurate information supplies ATC has to apply some necessary precautions while spacing the aircraft.

3 Capacity and delay models

In order to obtain an idea of the scope of currently existing analytical models to derive the airport runway capacity, some descriptions are given in the next sections. The difference between analytical and simulation capacity models is relatively simple. The former represent airport operations in an abstract mathematical form and the obtained expressions are manipulated either numerically or in closed form to derive estimates of capacity and delay. Analytical models are normally used to generate an early estimate of the effects of strategic decisions. A simulation capacity model consists of a very detailed description of all the elements involved. A large number of repetitions is generally required to obtain accurate estimates of capacity and delay. Since simulations are rather demanding both in terms of time and resources, they are normally used at a later stage, when detailed modelling of the problem becomes necessary. A review of general capacity and delay assessment tools is provided by Odoni $et\ al.$ in Ref. [21].

3.1 Capacity models

To determine the runway capacity at an airport, there are currently two analytical models in use. The first is the model proposed by Blumstein (Ref. [2]), of which a deviate is used as the building stone of the FAA Airfield Capacity Model (Ref. [26]). Although the Blumstein model was already developed some 40 years ago, it is still of interest because it essentially captures the main ideas in airport capacity modelling. The other is the more recently developed LMI Runway Capacity Model, which is integrated in the NASA Aviation System Analysis Capability airport capacity model (Ref. [16]). The LMI Runway Capacity Model was developed by the Logistics Management Institute (LMI) to overcome the limitations encountered in the Blumstein model. In a certain sense, the LMI model can be considered a direct extension to the Blumstein model. In the next sections a brief description of both the classical Blumstein and LMI runway capacity model is presented.

3.1.1 The Blumstein model

The Blumstein model considers a single runway used for arrivals only. The main assumptions are that the runway is operating to capacity, i.e. aircraft operate as close to each other as minimum separation permits, and that the aircraft maintain a constant velocity from the moment they enter the common approach path until they reach the runway threshold. Furthermore, arrivals are assumed to be independent and in random sequence. The runway capacity C is then defined as the number of flights this runway can accommodate during a certain unit of time. The chosen measure of capacity C is related to the average inter-arrival time between consecutive aircraft τ, through

$$C = \frac{1}{\tau}.$$

The inter-arrival times are obtained considering the separation distance minima imposed by ATC. Here τ can be determined from

$$\tau = \sum_{l,f} p_l p_f t_{lf},$$

where p_l and p_f are the relative frequencies by which aircraft of type l and f arrive at the airport and t_{lf} is the minimum time separation between a leading aircraft of type l and a following aircraft of type f at the runway threshold. The minimum time separation between two consecutive aircraft of types l and f is taken to be the maximum of the runway occupancy time for an aircraft of type l and the inter-arrival time between aircraft of types l and f when the separation distance constraint D_{lf} applies. The separation distance constraint can affect the inter-arrival time in two ways. If the speed V_l of the lead aircraft l is not greater than the speed V_f of following aircraft f, the distance constraint applies at the runway threshold. Then

$$t_{lf} = \max\left(R_l, \frac{D_{lf}}{V_f}\right),$$

where R_l indicates the runway occupancy time of a type l aircraft. If the speed V_l of the lead aircraft l is greater than the speed V_f of following aircraft f, the distance constraint applies at the beginning of the common approach path. In this case the time separation at the runway threshold is composed of the initial time separation and the time aircraft f loses to aircraft l during the approach. Now

$$t_{lf} = \max\left(R_l, \frac{S + D_{lf}}{V_f} - \frac{S}{V_l}\right),$$

where S indicates the length of the common approach path.

The Blumstein model provides a simple, yet effective, method for determining the landing capacity of a single runway. Within the FAA Airfield Capacity Model the imprecision of the aircrafts position is additionally incorporated in the Blumstein model. This is done by means of adding a stochastic variable to the separation distance minimum in such a way that the probability of violating the separation requirement is of level α. Stated in the form as described here, there seems to be no clear stochastic extension for all the other parameters.

3.1.2 The LMI runway capacity model

The main contribution of the LMI runway capacity model to the airport runway capacity modelling problem is that it provides a governing framework to link the occurring processes by taking a 'controller-based' view of airport operations, i.e. it is assumed that a person controls the aircraft in such a manner that all applicable rules are met with a specified level of confidence. As limitations on the quality of information accessible to this traffic controller directly affect the spacing required for a safe operation of aircraft streams, this yields a quite natural approach. As with the Blumstein model it is assumed that the aircraft maintain a constant velocity and operate as close to each other as made possible by the available information on typical aircraft characteristics. Again two separate cases based on the mean approach speeds of both the leading and the following aircraft are distinguished. The parameters considered in the LMI arrivals only module are listed in Table 2.

S	Length of common approach path
D_{ij}	Distance separation minimum for an aircraft of type i followed by an aircraft of type j
p_i	Fraction of aircraft that are of type i
V_i	Approach speed of aircraft i
dV_i	Variation in approach speed of aircraft i
dW_i	Variation experienced by aircraft i
dX_i	Position uncertainty of aircraft i
R_i	Arrival runway occupancy time of i^{th} aircraft
dR_i	Variation in R_i

Table 2 The LMI airport runway capacity modelling parameters.

It is assumed that each of the respective dV_i, dW_i, dX_i and dR_i variables are independent, random and that they follow a normal density with mean zero and a variance of $\sigma^2_{V_i}$, $\sigma^2_{W_i}$, $\sigma^2_{X_i}$ or $\sigma^2_{R_i}$ as appropriate.

Consider first the case in which the mean approach speed of the following aircraft f exceeds that of the leading aircraft l. Here it is assumed that the distance separation requirement D_{lf} applies as the leading aircraft crosses the runway threshold. At that time, the leader's position is S. Now $X_l(t)$, given by

$$X_l(t) = dX_l + (V_l + dV_l + dW_l)t,$$

denotes the position of the leading aircraft at time t on the common approach path. The leader then crosses the runway threshold at time t_l simply given by

$$t_l = \frac{S - dX_l}{V_l + dV_l + dW_l}.$$

At that moment in time the following aircraft's position $X_f(t_l)$ is given by

$$X_f(t_l) = dX_f + (V_f + dV_f + dW_f)\left(\frac{S - dX_l}{V_l + dV_l + dW_l} - \delta_t\right).$$

Here, δ_t denotes the time interval provided by the controller in order to assure that, as the lead aircraft crosses the runway threshold, the following aircraft is located at least a distance D_{lf} away from the runway threshold with a certain probability. To keep the problem tractable it is assumed that all disturbances are of first order and the result is made linear. This yields

$$X_f(t_l) = dX_f + \frac{V_f S}{V_l}\left(1 + \frac{dV_f + dW_f}{V_f} - \frac{dX_l}{S} - \frac{dV_l + dW_l}{V_l}\right) - V_f\left(1 + \frac{dV_f + dW_f}{V_f}\right)\delta_t.$$

Given this linear approximation $X_f(t_l)$ is a normal random variable of mean

$$\mathrm{E}X_f(t_l) = \frac{V_f S}{V_l} - V_f\delta_t$$

and variance

$$\mathrm{Var}X_f(t_l) = \sigma^2_{X_f} + \frac{(V_f S)^2}{V_l^2}\left(\frac{\sigma^2_{V_f} + \sigma^2_{W_f}}{V_f^2} + \frac{\sigma^2_{X_l}}{S^2} + \frac{\sigma^2_{V_l} + \sigma^2_{W_l}}{V_l^2}\right) + (\sigma^2_{V_f} + \sigma^2_{W_f})\delta_t^2.$$

The time interval δ_t complies with safety regulations based on a distance separation constraint if $S - X_f(t_l) \geq D_{lf}$. The condition that $S - X_f(t_l) \geq D_{lf}$ with probability at least 95 %, may then be stated as

$$\mathrm{E}X_f(t_l) + 1,65\sqrt{\mathrm{Var}X_f(t_l)} \leq S - D_{lf}.$$

Straightforward manipulations then lead to an explicit solution for δ_t, which can be written as

$$\delta_t = \frac{A + \sqrt{A^2B^2 + C^2(1 - B^2)}}{1 - B^2},$$

where

$$A = \frac{S}{V_l} - \frac{S - D_{lf}}{V_f},$$
$$B^2 = 1,65^2\left(\frac{\sigma^2_{V_f} + \sigma^2_{W_f}}{V_f^2}\right)$$

and

$$C^2 = \frac{1,65^2}{V_f^2}\left[\frac{V_f^2 S^2}{V_l^2}\left(\frac{\sigma^2_{V_f} + \sigma^2_{W_f}}{V_f^2} + \frac{\sigma^2_{X_l}}{S^2} + \frac{\sigma^2_{V_l} + \sigma^2_{W_l}}{V_l^2}\right) + \sigma^2_{X_f}\right].$$

Similarly, the time interval δ_t should guarantee that the following aircraft does not cross the runway threshold until the leading aircraft has left the runway at an exit. The lead aircraft will leave the runway at time

$$t_{R_l} = t_l + R_l + dR_l$$

whereas the follower will cross the threshold at time t_f given by

$$t_f = \frac{S - dX_f}{V_f + dV_f + dW_f} + \delta_t.$$

Linearizing as before $t_f - t_{R_l}$ is approximately a normal random variable with mean

$$\mathrm{E}(t_f - t_{R_l}) = \frac{S}{V_f} + \delta_t - \frac{S}{V_l} - R_l$$

and variance

$$\mathrm{Var}(t_f - t_{R_l}) = \frac{S^2}{V_f^2}\left(\frac{\sigma^2_{X_f}}{S^2} + \frac{\sigma^2_{V_f} + \sigma^2_{W_f}}{V_f^2}\right) + \frac{S^2}{V_l^2}\left(\frac{\sigma^2_{X_l}}{S^2} + \frac{\sigma^2_{V_l} + \sigma^2_{W_l}}{V_l^2}\right) + \sigma^2_{R_l}.$$

The condition on δ_t for the follower, such that it will not have crossed the runway threshold until the runway is vacant is that $t_f - t_{R_l} > 0$. It follows that this condition is satisfied with a probability of 98.7 % if

$$\delta_t \geq \frac{S}{V_l} - \frac{S}{V_f} + R_l + 2,215\sqrt{\mathrm{Var}(t_f - t_{R_l})}.$$

The controller will impose that value of δ_t that is the smallest to satisfy both the distance separation and the runway occupancy constraint. Given this δ_t, the time between the successive threshold crossings of consecutive arrivals t_{lf}, is, again in approximation, a normal random variable of mean

$$\mathrm{E}(t_{lf}) = \frac{S}{V_f} - \frac{S}{V_l} + \delta_t$$

and variance

$$\mathrm{Var}(t_{lf}) = \frac{S^2}{V_f^2}\left(\frac{\sigma^2_{X_f}}{S^2} + \frac{\sigma^2_{V_f} + \sigma^2_{W_f}}{V_f^2}\right) + \frac{S^2}{V_l^2}\left(\frac{\sigma^2_{X_l}}{S^2} + \frac{\sigma^2_{V_l} + \sigma^2_{W_l}}{V_l^2}\right).$$

Next, consider the case in which the follower's approach speed is slower than the leader's speed. This is handled differently. It is now assumed that the controller manoeuvres the follower such that it enters the common approach path after the leader has advanced a distance D_{lf} along it. Now the separation distance requirement is that $X_l(\delta_t) - X_f(\delta_t) \geq D_{lf}$. As

$$X_l(\delta_t) - X_f(\delta_t) = dX_l + (V_l + dV_l + dW_l)\delta_t - dX_f$$

is a normal random variable of mean

$$\mathrm{E}(X_l(\delta_t) - X_f(\delta_t)) = V_l\delta_t$$

and variance

$$\mathrm{Var}(X_l(\delta_t) - X_f(\delta_t)) = \sigma^2_{X_l} + (\sigma^2_{V_l} + \sigma^2_{W_l})\delta_t^2 + \sigma^2_{X_f},$$

it follows that the requirement on the separation distance is met with a probability of at least 95% if

$$V_l\delta_t + 1,65\sqrt{\sigma^2_{X_l} + (\sigma^2_{V_l} + \sigma^2_{W_l})\delta_t^2 + \sigma^2_{X_f}} \geq D_{lf}.$$

Straightforward manipulations then again lead to an explicit solution for δ_t, which can be written as

$$\delta_t = \frac{A + \sqrt{A^2B^2 + C^2(1 - B^2)}}{1 - B^2},$$

where

$$A = \frac{D_{lf}}{V_l},$$
$$B^2 = 1,65^2\left(\frac{\sigma^2_{V_l} + \sigma^2_{W_l}}{V_l^2}\right)$$

and

$$C^2 = 1,65^2\left(\frac{\sigma^2_{X_l} + \sigma^2_{X_f}}{V_l^2}\right).$$

The condition on the runway occupancy constraint is derived in exactly the same way as in the previous case and also the inter-arrival times mean and variance are of a similar form.

The statistic of the overall inter-arrival time is determined by considering the mix of aircraft using the runway and their respective values for the aircraft parameters. The previous results state the individual inter-arrival time t_{lf} for each aircraft pair l and f as a normal distributed random variable. Let the mean and variance for an aircraft of type l being followed by an aircraft of type f be μ_{lf} and σ^2_{lf} respectively. If $I_{l,f}$ denotes the indicator random variable corresponding to the aircraft landing sequence, the distribution function for overall inter-arrival time τ is given by

$$\tau = \sum_{l,f} I_{l,f} t_{lf}.$$

Obviously, τ is then a mixture of normal densities. If the arrivals are assumed to be independent and in random sequence, the inter-arrival time mean is given by

$$\mathrm{E}(\tau) = \sum_{l,f} p_l p_f \mu_{lf}$$

and the variance by

$$\mathrm{Var}(\tau) = \sum_{l,f} p_l p_f (\sigma^2_{lf} + \mu^2_{lf}) - \left(\sum_{l,f} p_l p_f \mu_{lf} \right)^2 .$$

The working definition of capacity C within the LMI model is taken to be

$$C = \frac{3600}{\mathrm{E}(\tau)}$$

arrivals per hour.

The description of the LMI Runway Capacity Model provided here closely follows the one given in Ref. [16]. Some modifications regarding parameter names have been made to achieve uniformity with the notation used in this paper.

3.2 Delay models

In the modelling of any form of delay a natural choice would be the use of queuing models. Queuing theory offers an enormous variety of models taking account of different server types, customer types and service disciplines. Within this context a customer requesting service can be an aircraft ready to land, the server can be either the runway or a part of the final approach path and the service time is the time needed to process the arrival. The esteemed delay, as experienced by arriving flights within the ATM system, obtained can then provide a measure of the airport's efficiency.

3.2.1 Transient queuing models

As Odoni and Roth (Ref. [19]) as well as Green and Kolesar (Ref. [9]) convincingly point out, the approximation of time-varying demand by the use of conventional steady state queuing models is highly inappropriate for the demand variation encountered at major hub airports. On accepting this, one resorts to non-stationary queuing systems. As it is only possible to calculate the exact probability distributions for some of the simpler transient queuing models, these systems are more difficult to evaluate and thus for the more general models other approaches are needed. Possible methods include fluid approximations as for instance described by Chen and Mandelbaum (Ref. [4]), closure approximations as described by Rothkopf and Oren (Ref. [24]), Glassey and Seshadri (Ref. [8]) and diffusion process approximations.

3.2.2 Non-stationary airport delay models

Work on airport runway capacity combined with some non-stationary queuing method was first published in 1958 by Galliher and Wheeler (Ref. [7]), who proposed the use of the $M(t)/D(t)/1$ queuing system to describe the landing process of congested aircraft. In 1972 Koopman (Ref. [14]) extended this approach by recommending the use of numerical solutions of the $M(t)/D(t)/1$ model's difference equations and the $M(t)/M(t)/1$ model's differential equations as lower and upper bounds on the behaviour of aircraft queues as experienced by airports. With a model based on these simple principles, a capacity study was undertaken by Odoni and Simpson (Ref. [20]) in 1975 to estimate the airfield capacity and the effects of the proposed fifth runway for Schiphol Airport. On modelling airport runway capacity in combination with some weather dependency a recent method developed by Peterson, Bertsimas and Odoni (Ref. [23]) proposes a Markov/semi-Markov treatment of changes of weather. Another approach is provided by Abundo (Ref. [1]), who proposes a combination of a $M(t)/E_k(t)/1$ model for the landing queue with a simulation of an airport's capacity profile over time.

4 Model considerations

It seems that the main task in determining the airport airside capacity is to obtain a satisfactory manner for converting the required longitudinal wake turbulence spacing into separation times and then compare these with the respective time needed to fulfil runway occupancy constraints. While in its own this seems to yield a process interesting enough to investigate, prior considerations have to be made before further examination of the subject.

4.1 Definitions

Suppose it were possible to unambiguously provide a concise description of what is actually meant when referring to *capacity*. Such a definition would then immediately give rise to questions on which measures should be taken to create those circumstances beneficial to optimally achieve this capacity. In airport runway capacity modelling, capacity is usually described as being the number of aircraft operations performed in a period of time. An alternative definition in use, is the number of aircraft operations that can be performed in a period of time with a prescribed level of confidence. When certain prior assumptions are made, both these capacity metrics can then be determined by considering sequences of operations of arbitrary length. However, none of these descriptions provide a satisfactory answer to the question remarked before. In both cases optimal capacity would only be reached under the circumstances that there would always be a next aircraft ready to land, i.e. when there is at least one, up to infinitely many aircraft awaiting landing clearance. Since it is highly unlikely that every single aircraft appears within the ATM system exactly when safety restrictions allow so, capacity and delay are obviously related. In a sense delay, for example measured as average delay per operation, is the price paid for achieving capacity. With this in mind a better expression for the amount of operations per unit time would be the *service rate*. This would then leave the term capacity open to refer to the number of operations per unit time for a given assumption on delay. This alternative description of capacity would then consist with the interrelated nature of capacity and delay.

4.2 Capacity

In addition to the factors influencing airport capacity mentioned before, and in regards to the methods to derive the runway capacity described in sections 3.1.1 and 3.1.2, some aspects of influence that are perhaps not accounted for to their fullest potential deserve some extra thoughts.

4.2.1 Aircraft motion

In both the Blumstein and the LMI runway capacity model the aircraft's speed is assumed to remain constant on the entire common approach path. However, Figure 2 not only depicts the rather exotic diversity of aircraft speeds in the course of the runway approach for a common aircraft

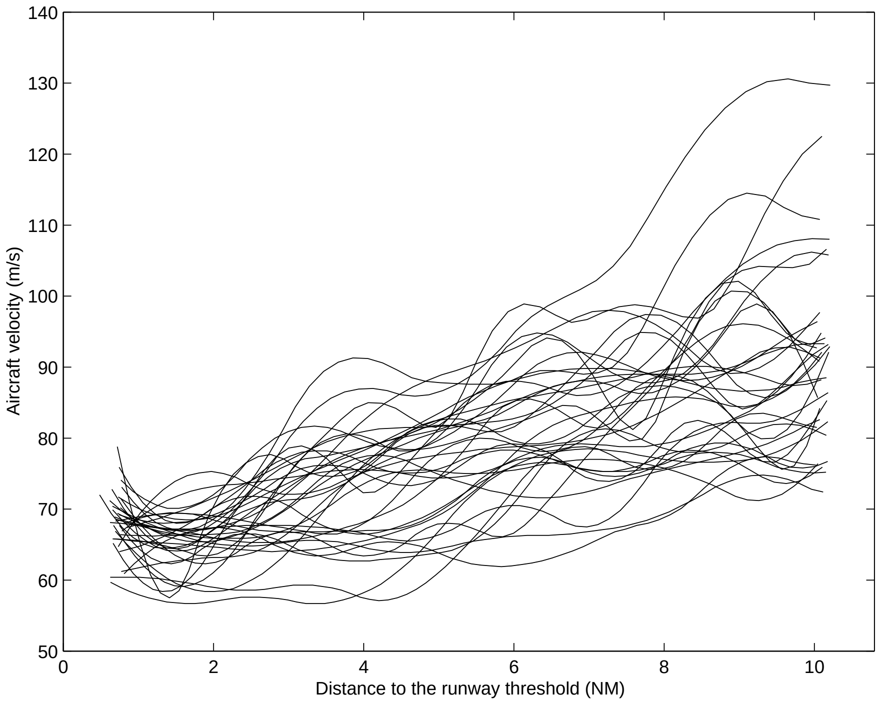

Fig. 2 Variation in approach speed for a single aircraft type. Measurements consist of a single day of B737-300 arrivals for Schiphol Airport runway 19R as recorded by FANOMOS on April 3rd 1999.

type, it also shows a more general aircraft tendency to gradually decelerate while closing in on the runway threshold. In view of runway capacity changes as an immediate consequence of either reduced or increased separation distances, any attempt without taking this aircraft deceleration into account can be considered in vain. In a model not covering this aircraft motion, capacity would for instance increase linearly with a decrease in required spacing. In practice however, due to the aircraft deceleration, the reduced separation distance is covered at a lower speed.

4.2.2 Air traffic control method

Another aspect that should be highlighted is that in ATC daily practice, all sequenced aircraft are supposed to be properly spaced as the lead aircraft crosses some nominated point, for example the runway's outer marker. This can be understood when simply considering aircraft that actually decelerate. Then, rather independent of which aircraft types are longitudinally spaced in the course of a runway approach and their sequence, the rear aircraft shall generally be in a different phase of flight and will tend to have a greater speed. Due to this higher velocity any following aircraft shall, perhaps only temporarily, gain some distance on its predecessor. With this in mind, considering separate spacing cases for quicker or slower aircraft, as done in both the Blumstein and the LMI

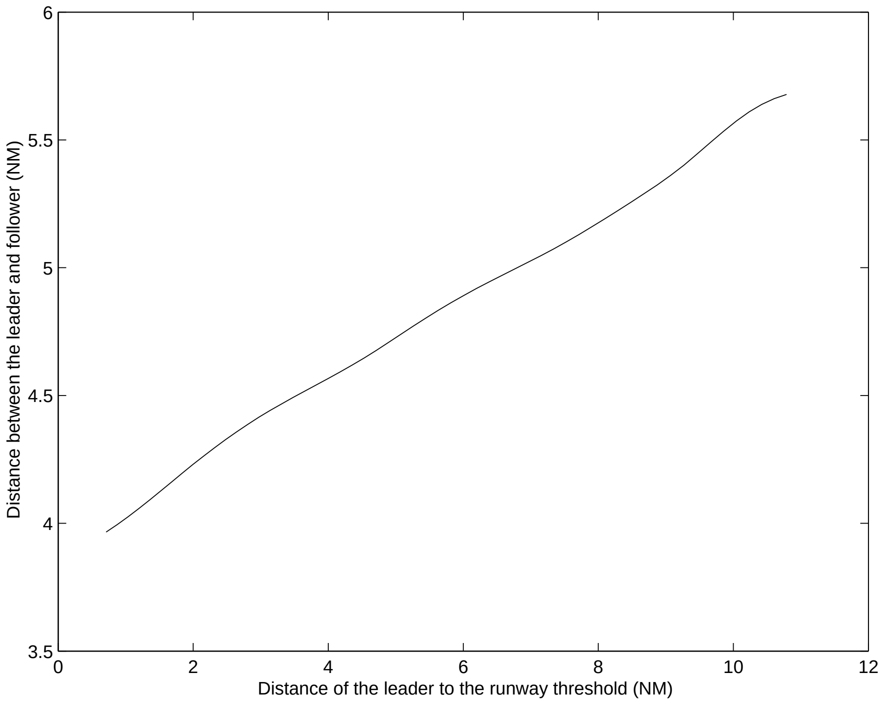

Fig. 3 Actual separation between a B767-300 (HEAVY) and a following B747-400 (HEAVY) arriving on Schiphol Airport runway 06 as obtained by inspection of FANOMOS recorded track measurements in the early morning of April 3^{rd} 1999. The outer marker is located almost 4NM off the runway threshold.

runway capacity model, seems rather odd. To illustrate the actual longitudinal spacing principle, Figure 3 depicts the actual separation between two consecutive arriving aircraft. Of course given the aircraft's deceleration pattern, the location of such a fix where the required separation distance is to be guaranteed is crucial in determining the actual landing capacity of a runway.

4.3 Delay

Whereas the remarks made in the previous section on capacity consist of limitations perhaps posed by restrictive modelling, the remarks made in this section directly relate to constraints posed by the delay model itself.

4.3.1 Variation in service rates

On studying the queuing models used for delay approximation, a rather odd observation is quickly made: although every delay model stresses the necessity of a time varying demand process, the fact that the service times also tend to vary in the course of the day seems completely overlooked. Perhaps because the shifts in the traffic mix produce rather subtle effects compared to the abrupt jolts in the traffic demand profile.

In view of this papers purpose, introducing a weather dependent runway capacity model, thus multiple service rates are to be considered anyway, ignoring the shifts in the traffic mix can be considered as rather easygoing, but there are insurmountable theoretical objections against not doing so. The problem is found in the moment at which the service time for a certain customer is determined. This can either be the moment the customer arrives at the end of the queue or the actual moment when its service starts. These both instants of service time determination yield entirely different queuing models. A change in traffic mix essentially involves a change of customer type. This is hence preferably taken into account at the moment the new customer enters the arrival stack. A change in separation minima generally comes down to a different server type, preferably taken account of at the exact moment this new server commences activity. When considering changes in service rate due to both shifts in traffic mix and weather dependent minimum spacing, only one of these effects can be properly modelled. Hence a model that is not explicitly making this distinction shall be prone to theoretical errors. On the other hand, a model that does distinguish these sources of variability, may provide false delay estimates, as generally the arrival peaks are coupled to the worst possible traffic mix proportions in terms of runway capacity.

When simply ignoring shifts in the traffic mix is not considered an option, a choice between the two queuing models needs to be made. Assume that during a certain period a stack of aircraft is created and that service times are determined when arriving at the stack. Given these circumstances no problems are encountered when the traffic mix changes for every aircraft in stack is served under the traffic mix premises valid in the period it arrived. However, the prescribed spacing on arrival at the stack may very well differ to that required when actually landing. This may result in either overly conservative or unsafe aircraft spacing. Although both of these effects are unwanted, based on the latter, this situation is deemed unacceptable. For the other type of models, a change in minimum spacing may occur instantly, though it can be assumed that this was planned well in advance, as the first aircraft is being served under new turbulence separation premises. However, with regards to shifts in the traffic mix, also the aircraft composition of the stack literally changes instantly. While this seems less a problem, some loss of information can be expected. In this respect allowing a variable traffic mix, given a non-stationary queuing model, necessarily involves a choice between two evils.

4.3.2 Interdependence of service rates

Another effect related to the traffic mix is the interdependence of the service rates. As in most non-stationary queuing systems the service rates are assumed to be stochasticly independent, the best guess here is to use such a service times distribution that, while every sequence of outcomes is completely random, in terms of a queuing system approximately behaves like the sequenced aircraft reality on the long run. Despite the fact that this restriction also introduces errors, modelling the dependence, at the level of accounting for the types of aircraft involved in successive landings, would require a model of a far more microscopic nature than needed for the purposes here.

5 Proposed model

5.1 Capacity module

Adopting the LMI model's 'controller-based' view to aircraft operations, it is assumed that given a certain distance separation requirement a controller manoeuvres the following aircraft such that it enters the common approach path a time δ_t behind the leading aircraft. This time separation δ_t is chosen in light of knowledge of typical approach speeds and disturbances on the relative longitudinal positions of both aircraft, such that the separation requirement and runway occupancy rules are met with a certain level of confidence.

5.1.1 Distance separation requirement

Depending on the ATC policy a distance separation usually applies at some fixed point in respect to the runway threshold, for instance the runway threshold itself or the runway's outer marker. As a separation requirement at an outer marker provides a more general view to the modelled process, it will be discussed here. Consider the case of a leading aircraft l and a following aircraft f. Let the origin of the co-ordinate axis along the common approach path be at the runway threshold. Then let S denote the distance to the runway threshold at the beginning of the common approach path and the location of the outer marker is indicated by O.

Method outline

Assume that both the leading and the following aircraft's actual speed $V_l(x)$ and $V_f(x)$ as functions of x, the distance to the runway threshold are as depicted in Figure 4. Then given the moment in time t_0, when the leading aircraft l enters the common approach path at S, the easiest notion of the moment in time when aircraft l will eventually reach the outer marker is obtained through using V_{lo}, l's average speed travelling from the beginning of the approach path towards the outer marker. Given these variables the position of the leading aircraft relative to the runway threshold as a function of time on this initial part of the approach can be modelled by

$$X_{lo}(t) = S - V_{lo}(t - t_0).$$

This function is highly inaccurate in describing the aircraft's actual movement. It does however, provide some information on the sole two moments that matter within this context. Without loss of generality suppose that the first of these moments $t_0 = 0$. Then elementary calculations yield the second instant

$$t_{lo} = \frac{S - O}{V_{lo}},$$

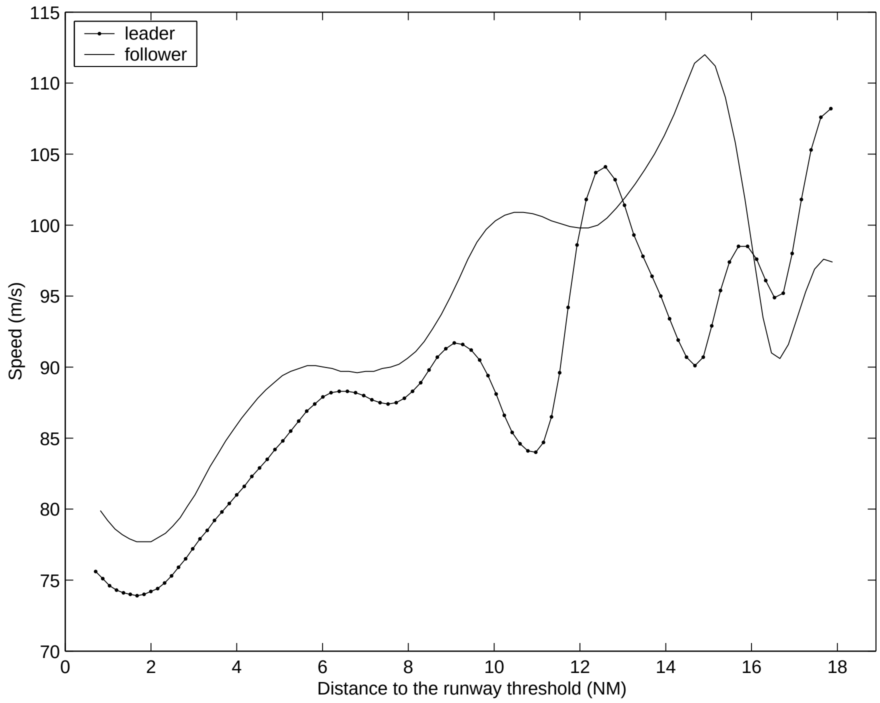

Fig. 4 Approach speeds for a leading B767-300 (HEAVY) and a following B747-400 (HEAVY) arriving on Schiphol Airport runway 06 as obtained by FANOMOS recorded track measurements in the early morning of April 3^{rd} 1999.

the moment the leader crosses the outer marker. On appropriately separating the following aircraft f the problem is to determine the minimum spacing in time δ_t such that when f enters the common approach path at $t = \delta_t$, it will not have reached the separation point $O + D_{lf}$ before l crosses the outer marker. Here D_{lf} is simply the required separation minimum for a leading aircraft of type l and a following aircraft of type f, as prescribed by ATC regulations. Again the easiest method is by similarly using the following aircraft's average speed when flying from the beginning of the common approach path towards the separation point. Denote this average speed by V_{fs} . Then δ_t is given by

$$\delta_t = \frac{S - O}{V_{lo}} - \frac{S - (O + D_{lf})}{V_{fs}}.$$

Arguing analogously, given both the leading and the following aircraft's average speeds on the complete approach path V_l and V_f and the time separation δ_t between l and f at the beginning of the approach path S, the time t_{lf} passed between these consecutive arrivals over the runway threshold is then simply given by

$$t_{lf} = \frac{S}{V_f} - \frac{S}{V_l} + \delta_t.$$

Note that, perhaps upon a second inspection of the argumentation followed up till now, there is an odd twist with regards to the aircraft speeds used. Take the reasoning on V_{lo} for instance. As the only method to obtain V_{lo} out of $V_l(x)$ is through

$$V_{lo} = \frac{S - O}{\int_O^S \frac{dx}{V_l(x)}},$$

computation of V_{lo} actually involves evaluation of t_{lo}. This distortion is however for the moment kept in deliberately to emphasize, by means of a similar notation both the resemblances and the differences with already existing runway capacity models. Also both functions $V_l(x)$ and $V_f(x)$ used thus far clearly represent arbitrary outcomes of a runway approach process for certain types of aircraft. Unfortunately, like in any other stochastic process, these are not known in advance and, as everything is pretty obvious at hindsight, therefore not immediately relevant in modelling the situation. To overcome this problem one can consider average profiles of what the speed of certain aircraft types typically behaves like and an attempt can be made to extend the approach such that as many arbitrary speed outcomes are included by means of suitably chosen confidence intervals.

Method extension with stochastic perturbations

Suppose for a moment that speeds as V_l, V_{lo} for a leading aircraft and their follower counterparts V_f and V_{fs} can be obtained out of a sample of tracks $V(x)$ for a certain aircraft class. Out of such analysis also dV_l, dV_{lo}, dV_f and dV_{fs} can be estimated. These variables emerge as the stochastic perturbations to their respective averages V_l, V_{lo}, V_f and V_{fs}. It should be noted that such direct estimations of these uncertainties may be heavily contaminated by ATC speed warrants and perhaps do not simply represent variation on the basis of aircraft performance on final approach only. It is assumed that all perturbations are independent, and follow normal distributions of mean zero and variance $\sigma^2_{V_l}$, $\sigma^2_{V_{lo}}$, $\sigma^2_{V_f}$ and $\sigma^2_{V_{fs}}$ as appropriate. In addition to extending towards varying approach speeds, the moment the leading aircraft enters the common approach path t_0 cannot be considered deterministic due to various factors. However, this can easily be reversed: suppose that at a known moment in time t_0 the leading aircraft l is positioned at a location $S + dS_l$ from the runway threshold. It is assumed here that dS_l depends solely on the ATC's radar frequency R_f and the aircrafts average speed on location S: $V_l(S)$. Let $R_t = R_f^{-1}$ denote the time between the refreshments of radar information. It can be argued that any position somewhere between $S \pm R_t V_l(S)$ can be accepted as being S. Under these assumptions the best guess at the distribution of these positions would probably be a uniform $\mathcal{U}(S - R_t V_l(S), S + R_t V_l(S))$ distribution. This however, does not take into account the attempts of ATC to perform their job properly, nor is the uncertainty in the aircrafts speed at which it arrives at S, i.e. $dV_l(S)$ included. To keep the problem tractable, dS_l is assumed to have a normal distribution with the same variance as the uniform guess before. That is: $dS_l \sim \mathcal{N}(0, \frac{(R_t V_l(S))^2}{3})$.

Given these assumptions, the position of the leading aircraft relative to the threshold as a function of time on the initial part of the approach can be modelled by

$$X_{lo}(t) = S + dS_l - (V_{lo} + dV_{lo})(t - t_0).$$

Now assuming $t_0 = 0$, then

$$t_{lo} = \frac{S - O + dS_l}{V_{lo} + dV_{lo}}$$

provides the moment the leader crosses the outer marker. With the same arguments as before, now the entrance position of the following aircraft f is assumed to be S with uncertainty $dS_f \sim \mathcal{N}(0, \sigma^2_{S_f})$. As this aircraft now enters the approach path δ_t seconds behind the lead aircraft, its position function is now given by

$$X_{fs}(t) = S + dS_f - (V_{fs} + dV_{fs})(t - \delta_t).$$

The crucial position to determine the yet unknown δ_t with, is again obtained by substituting $t = t_{lo}$. Simple rewriting then gives

$$X_{fs}(t_{lo}) = S + dS_f - \left(\frac{V_{fs}(S-O)}{V_{lo}}\delta_t\right)\left(1 + \frac{dV_{fs}}{V_{fs}}\right)\left(\frac{1 + \frac{dS_l}{S-O}}{\delta_t\left(1 + \frac{dV_{lo}}{V_{lo}}\right)} - \frac{V_{lo}}{S-O}\right).$$

When the variance of the disturbance to V_{lo} allows so, the following approximation can be used in order to keep the problem tractable.

$$\left(1 + \frac{dV_{lo}}{V_{lo}}\right)^{-1} \approx 1 - \frac{dV_{lo}}{V_{lo}}$$

Substituting this approximation and linearizing the obtained equation simply yields the following representation of $X_{fs}(t_{lo})$, as approximately a normal distributed random variable

$$X_{fs}(t_{lo}) \approx S + dS_f - \left(\frac{V_{fs}(S-O)}{V_{lo}}\right)\left[\left(1 - \frac{V_{lo}\delta_t}{S-O}\right)\left(1 + \frac{dV_{fs}}{V_{fs}}\right) + \left(\frac{dS_l}{S-O} - \frac{dV_{lo}}{V_{lo}}\right)\right]$$

with mean

$$\mathrm{E}X_{fs}(t_{lo}) = S - \frac{V_{fs}(S-O)}{V_{lo}} + V_{fs}\delta_t$$

and variance

$$\mathrm{Var}X_{fs}(t_{lo}) = \sigma^2_{S_f} + \left(\frac{S-O}{V_{lo}} - \delta_t\right)^2 \sigma^2_{V_{fs}} + \left(\frac{V_{fs}(S-O)}{V_{lo}}\right)^2 \left(\frac{\sigma^2_{S_l}}{(S-O)^2} + \frac{\sigma^2_{V_{lo}}}{V^2_{lo}}\right).$$

Considering the stochastic nature of the components involved, a definition of a safe time separation at S can be a choice of δ_t such that

$$\mathrm{E}X_{fs}(t_{lo}) + 1,65\sqrt{\mathrm{Var}X_{fs}(t_{lo})} \geq O + D_{lf}.$$

With this definition the separation distance criterion is accomplished with 95% confidence. Straightforward manipulations now yield

$$\delta_t \geq \sqrt{\frac{B^2-A}{D} + \left(\frac{C}{D}\right)^2} + \frac{C}{D}$$

where

$$\begin{aligned}
A &= 1,65^2 \left[\sigma^2_{S_f} + \left(\frac{V_{fs}(S-O)}{V_{lo}}\right)^2 \left(\frac{\sigma^2_{S_l}}{(S-O)^2} + \frac{\sigma^2_{V_{fs}}}{V^2_{fs}} + \frac{\sigma^2_{V_{lo}}}{V^2_{lo}}\right)\right],\\
B &= O + D_{lf} - S + \frac{V_{fs}(S-O)}{V_{lo}},\\
C &= 1,65^2 \left(\frac{S-O}{V_{lo}}\right)\sigma^2_{V_{fs}} - 2\left(O + D_{lf} - S + \frac{V_{fs}(S-O)}{V_{lo}}\right)V_{fs}
\end{aligned}$$

and

$$D = 1,65^2\sigma^2_{V_{fs}} - V^2_{fs}.$$

Now reasoning analogously as before, the inter-arrival time t_{lf}, i.e. the time passed between the two consecutive arrivals over the runway threshold is then given by

$$t_{lf} = \frac{S + dS_f}{V_f + dV_f} - \frac{S + dS_l}{V_l + dV_l} + \delta_t.$$

Thus far the model logic merely consists of a natural generalization of a part of the LMI runway capacity model. Unfortunately, the reality of daily life does not directly allow the use of the simplifying approximations. The essential consequence of these approximations is that $X_{fs}(t_{lo})$ can conveniently be described in closed form as a normal distributed random variable. Simulation results show that $X_{fs}(t_l)$ does follow a normal distribution and that hence, the corresponding results still stand. However without considerable effort the model can be stated in a different, but equivalent form that does not rely on density approximations and lacks the possible errors hence introduced.

Proposed method

Consider the distance separation requirement as described previously. Almost every equation consists of some speed and the distance to which this speed is averaged. Let, for instance, instead of the combined effort of V_{lo} and $S - O$, T_{lo} simply denote the time on average needed by the lead aircraft l to travel the distance between the entry location on the common approach path and the runway's outer marker. Suppose this time T_{lo} is being distorted by $dT_{lo} \sim \mathcal{N}(0, \sigma^2_{T_{lo}})$, the variation to the average time and $dT_S \sim \mathcal{N}(0, \sigma^2_{T_S})$, the variation in time as a consequence of the uncertainty of the actual entry position on the approach path. Again it is assumed that all the introduced perturbations are independent of each other. Then t_{lo}, the moment the leading aircraft crosses the outer marker, is simply given by

$$t_{lo} = T_{lo} + dT_{lo} + dT_S.$$

With similar assumptions for the following aircraft f, the moment in time t_{fs} when the following aircraft reaches the separation point $O + D_{lf}$, where O is the location of the outer marker respective to the runway threshold and D_{lf} the required longitudinal spacing between a lead aircraft of type l and a follower of type f, is given by

$$t_{fs} = T_{fs} + dT_{fs} + dT_S + \delta_t.$$

Here T_{fs} and $dT_{fs} \sim \mathcal{N}(0, \sigma^2_{T_{fs}})$ are defined similarly as with the leading aircraft and δ_t is the time separation at the beginning of the common approach path, as imposed by the air traffic controller. Now $t_{fs} - t_{lo}$ is a normal distributed random variable of mean

$$\mathrm{E}(t_{fs} - t_{lo}) = T_{fs} - T_{lo} + \delta_t$$

and variance

$$\mathrm{Var}(t_{fs} - t_{lo}) = \sigma^2_{T_{fs}} + \sigma^2_{T_{lo}} + 2\sigma^2_{T_S}.$$

Then, in accordance to the safety definitions used previously, δ_t can be obtained by simply solving $t_{fs} - t_{lo} \geq 0$ at 95% confidence. This results in

$$\delta_t \geq T_{lo} - T_{fs} + 1,65\sqrt{\sigma^2_{T_{fs}} + \sigma^2_{T_{lo}} + 2\sigma^2_{T_S}}.$$

With this time separation between the consecutive aircraft at the beginning of the common approach path, the time passed between these two arrivals over the runway threshold t_{lf} is given by

$$t_{lf} = (T_f + dT_f + dT_S) - (T_l + dT_l + dT_S) + \delta_t,$$

where T_l and T_f are the respective average times to complete the approach and their respective perturbations dT_l and dT_f are again assumed to follow normal distributions with mean zero and variances $\sigma^2_{T_l}$ and $\sigma^2_{T_f}$ as appropriate.

5.1.2 Runway occupancy requirement

The amount of time an aircraft occupies the runway is a crucial factor restricting runway capacity. Suppose for a moment that no separation distance is required and that separation is provided solely on the basis of the availability of the runway. Assume that in a landing sequence the leading aircraft l on average occupies the runway for a period of R_l seconds. Here again the variation to the average runway occupancy time dR_l is assumed to be a normal distributed random variable, $dR_l \sim \mathcal{N}(0, \sigma^2_{dR_l})$. As before, the leading aircraft of the pair is assumed to enter the common approach path at $t_0 = 0$. With the same variables and notation as used in the previous section, the runway will be available for the following arrival f at $t = t_{R_l}$, where t_{R_l} is given by

$$t_{R_l} = T_l + dT_l + dT_S + R_l + dR_l.$$

As the following aircraft enters the approach path at S, the beginning of the common approach path, with a time separation of δ_t seconds behind the leading aircraft, it will reach the threshold at $t = t_f$, where

$$t_f = T_f + dT_f + dT_S + \delta_t.$$

Now the behaviour of $t_f - t_{R_l}$ determines whether the time separation δ_t can be considered sufficiently safe or potentially hazardous, as runway incursions are allowed too frequently. Here $t_f - t_R$ is a normal distributed random variable with mean

$$\mathrm{E}(t_f - t_R) = T_f - (T_l + R_l) + \delta_t$$

and variance

$$\mathrm{Var}(t_f - t_R) = \sigma^2_{T_f} + \sigma^2_{T_l} + \sigma^2_{R_l} + 2\sigma^2_{T_S}.$$

Solving $t_f - t_R \geq 0$ for δ_t at 95% confidence yields

$$\delta_t \geq T_l - T_f + R_l + 1,65\sqrt{\sigma^2_{T_f} + \sigma^2_{T_l} + \sigma^2_{R_l} + 2\sigma^2_{T_S}}$$

With this time separation δ_t between the two aircraft at the beginning of the common approach path, the time between the two arrivals over the runway threshold t_{lf} is is given by

$$t_{lf} = (T_f + dT_f + dT_S) - (T_l + dT_l + dT_S) + \delta_t,$$

which is of a similar form as before.

5.1.3 Runway capacity

According to safety regulations an aircraft can only be awarded ATC clearance to land if and only if both the restrictions to the distance separation and the runway occupancy requirement are satisfied. For both these criteria, the time passed between consecutive arrivals over the runway threshold, the inter-arrival time t_{lf}, is of the same form, in which δ_t represents the time separation needed at the beginning of the common approach path to ensure that, with a proper confidence level, the operation can be performed according to safety regulations. Let from now on, δ_t denote the smallest value that satisfies both the distance separation and the runway occupancy constraint and assume that, in fact, a time separation of $\delta_t + \epsilon_t$ seconds is imposed by ATC at the beginning of the common approach path. Here ϵ_t is a fixed time increment. As before it follows that t_{lf}, now given by

$$t_{lf} = (T_f + dT_f + dT_S) - (T_l + dT_l + dT_S) + \delta_t + \epsilon_t,$$

is a normal distributed random variable with mean μ_{lf} where

$$\mu_{lf} = T_f - T_l + \delta_t + \epsilon_t$$

and variance σ^2_{lf} given by

$$\sigma^2_{lf} = \sigma^2_{T_f} + \sigma^2_{T_l} + 2\sigma^2_{T_S}.$$

Assuming that the leading and following aircraft are independently drawn from the traffic mix, may result in an estimate of the overall inter-arrival time distribution τ. Since $t_{lf} \sim \mathcal{N}(\mu_{lf}, \sigma^2_{lf})$, it follows that τ is then distributed as a mixture of normal distributions, $\tau \sim \sum_{l,f} I_{l,f} t_{lf}$, where $I_{l,f}$ denotes the indicator random variable corresponding to the aircraft landing sequence. The inter-arrival time mean is then given by

$$\mathrm{E}(\tau) = \sum_{l,f} p_l p_f \mu_{lf}$$

and the variance by

$$\mathrm{Var}(\tau) = \sum_{l,f} p_l p_f (\sigma^2_{lf} + \mu^2_{lf}) - \left(\sum_{l,f} p_l p_f \mu_{lf} \right)^2.$$

Conversely, it can be assumed, and this is generally the case, that the sequence of aircraft depends

on the actual types and hence classification, as the ATC's effort is particularly directed towards optimisation of the arrival stream. Although there is a limit to the sequencing results achievable, the expenses paid, very well result in a better back to back positioning of the heavy class aircraft and a more coherent flow of aircraft in terms of speed differences. As the work done thus far consists of evaluation of the inter-arrival times for pairs of aircraft, a reasonable alternative to the occurrence probability of a lead aircraft of type l and a follower aircraft of type f is through using p_{lf}, the probability that these aircraft are sequenced in practice. This would not alter the form of the expressions on $\mathrm{E}(\tau)$ or $\mathrm{Var}(\tau)$ considerably. What is readily changed is that these statements would then also provide a measure to the managing accomplishments on the ATC's account. Note that given the nice behaviour of the inter-arrival time t_{lf} for the respective aircraft pair, one can easily evaluate other interesting distributions that bear some similarity to τ. For instance, if additionally a Markov property is assumed for arriving aircraft pairs, which, stated more commonly means that the fact that the lead aircraft is actually also a follower to another aircraft is assumed to have no influence on the arrival processes for both respective aircraft pairs, sequences of landing aircraft of arbitrary length can be considered. As much of the work so far has been rather approximate, the hourly capacity C can very well be defined as

$$C = \frac{3600}{\mathrm{E}(\tau)}$$

and this definition is thus solely based on the inter-arrival time between pairs of consecutive landing aircraft.

5.2 Delay module

In terms of queuing theory τ can be considered a service rate distribution. As mentioned before, the use of steady state queuing methods is not appropriate for the demand variation encountered at large hub airports. Despite the fact that the majority of queuing systems encountered in daily life probably behave transiently, the amount of work done on approximating non-stationary queuing systems is surprisingly small. In this regard, the realistic possibilities to approximate the inter-arrival time density are restricted to deterministic, Markov and Erlang service rate processes.

5.2.1 Erlang delay estimate

Assuming that aircraft arrive in the ATC's terminal area according to a Poisson process, intuitively, the best service rate distribution to consider seems the Erlang density. The standard $M/E_k/1$ queue is for instance described in Saaty Ref. [25] on p.164. When using the Erlang distribution E_k as service distribution, each customer entering can be considered to generate k phases of service. These phases have an identical exponential distribution with parameter $k\mu_k$. Then the service time distribution is given by

$$\frac{(k\mu_k)^k}{\Gamma(k)} e^{-k\mu_k t} t^{k-1}, \quad 0 \leq t < \infty.$$

Let $P_n(t)$ denote the probability that there are n phases waiting and in service at time t. Each aircraft arrival at the end of the queue increases the number of phases in the system by k and each phase completed decreases the number of phases by one. Then the Kolmogorov forward equations are given by

$$P_n(t + \Delta_t) = P_n(t)[1 - (\lambda + k\mu_k)\Delta_t] + P_{n+1}(t)k\mu_k\Delta_t + P_{n-k}(t)\lambda\Delta_t$$

for $n \geq 1$. Here λ indicates the arrival process parameter. The transient $M/E_k/1$ queue can be obtained by introducing a time dependency on λ and μ_k. This can easily be done by substituting $\lambda = \lambda(t)$ and $\mu_k = \mu_k(t)$. Letting $\Delta_t \to 0$ yields

$$P_n'(t) = \lambda(t)P_{n-k}(t) + k\mu_k(t)P_{n+1}(t) - (\lambda(t) + k\mu_k(t))P_n(t)$$

for $n \geq 1$. This results in an infinite number of differential equations describing the queuing system's evolution over time. Another Erlang distribution based approximation to the delay process is by means of considering the Erlang E_k^{μ} distribution for the service rates. Again each customer entering the system generates k phases of service, but now these phases have identical exponential distributions with parameter μ. Arguing as before the system of differential equations can now be derived from

$$P_n(t + \Delta_t) = P_n(t)[1 - (\lambda + \mu)\Delta_t] + P_{n+1}(t)\mu\Delta_t + P_{n-k}(t)\lambda\Delta_t$$

for $n \geq 1$.

As pointed out by Koopman in Ref. [14], imposing a maximum upper limit m on the queue length avoids theoretical difficulties, and is realistic in terms of the practice at airports where aircraft are sometimes diverted as stack space is limited. Moreover, when using this simple truncation, the number of aircraft diverted under various conditions can be determined. As both these models now become more and more promising, the possible variability in runway capacity as a result of varying distance separation requirements through time spoils the game. To see this, consider simple moment estimates to the Erlang parameters for both cases. The estimate for μ_k is then given by

$$\mu_k = \frac{1}{\mathrm{E}(\tau)},$$

the estimate for μ is given by

$$\mu = \frac{\mathrm{E}(\tau)}{\mathrm{Var}(\tau)}$$

and for both systems k by

$$k = \left[\frac{\mathrm{E}(\tau)^2}{\mathrm{Var}(\tau)}\right],$$

which does not exclude the possibility that, in fact a proper estimate of k should also be allowed to vary through time. There is no clear solution to this problem. Fortunately, a queuing system generally accounted to Koopman (Ref. [14]) provided, at least in the numerous fixed k cases ran, surprisingly well results compared to the actual Erlang systems, and this approximating queuing model does not bear such deficiencies when extended to varying service rates.

5.2.2 Deterministic delay estimate

The system provided by Koopman in Ref. [14] can in fact be dated as far back to 1932 when it was first published in some form by Crommelin (Ref. [6]).

Queue length approximation

It is assumed that, the time passed between two consecutive landing aircraft is exactly $\mu(t)$ units of time. This results in a sequence of points $t_1, t_2, t_3, \ldots$ in time, spaced $\mu(t_i)$ apart that represent the landing instants. Hence, the attention is restricted to only those points in this sequence. For landing aircraft this means that, should the queue contain a positive number of aircraft, one aircraft is removed at such instants t_i. For aircraft arriving at the end of the queue it is assumed they will, if the queue length allows so, be admitted to the queue at the next instance of evaluation. The probability that a number of n aircraft attempt to join the queue at t_{i+1} is given by the Poisson expression $(a_i^n/n!)e^{-a_i}$, where $a_i = \mu(t_i)\lambda(t_i)$ and $\lambda(t_i)$ is the arrival parameter of the Poisson process if $t = t_i$. Suppose the maximum aircraft queue length is restricted to m stack spaces and let p_j^i denote the probability of j aircraft in the system at $t = t_i$. Then p^{i+1} can be obtained out of p^i by solving

$$
\begin{aligned}
p_0^{i+1} &= (p_0^i + p_1^i)e^{-a_i} \\
p_1^{i+1} &= (p_0^i + p_1^i)a_i e^{-a_i} + p_2^i e^{-a_i} \\
p_2^{i+1} &= (p_0^i + p_1^i)(a_i^2/2!)e^{-a_i} + p_2^i a_i e^{-a_i} + p_3^i e^{-a_i} \\
&\vdots \\
p_n^{i+1} &= (p_0^i + p_1^i)(a_i^n/n!)e^{-a_i} + p_2^i[a_i^{n-1}/(n-1)!]e^{-a_i} + \cdots + p_{n+1}^i e^{-a_i} \\
&\vdots \\
p_m^{i+1} &= (p_0^i + p_1^i)u_m^i + p_2^i u_{m-1}^i + \cdots + p_m^i u_1^i
\end{aligned}
$$

where u_n^i is given by

$$\sum_{j=n}^{\infty}(a_i^j/j!)e^{-a_i} = 1 - \sum_{j=0}^{n-1}(a_i^j/j!)e^{-a_i}$$

In actual numerical computation this system of recurrence equations can be replaced by a slightly modified system. When introducing a new symbol $\bar{p}_1^i$, defined by $\bar{p}_1^i = p_0^i + p_1^i$,the recurrence relations can be written in matrix form as

$$
\begin{bmatrix} \bar{p}_1^{i+1} \\ p_2^{i+1} \\ \vdots \\ p_{m-1}^{i+1} \\ p_m^{i+1} \end{bmatrix}
=
\begin{bmatrix}
(1+a_i)e^{-a_i} & e^{-a_i} & 0 & 0 & \cdots & 0 \\
(a_i^2/2!)e^{-a_i} & a_i e^{-a_i} & e^{-a_i} & 0 & \cdots & 0 \\
\vdots & \vdots & & & & \vdots \\
(a_i^m/m!)e^{-a_i} & [a_i^{m-1}/(m-1)!]e^{-a_i} & & & \cdots & e^{-a_i} \\
u_m^i & u_{m-1}^i & & & \cdots & u_1^i
\end{bmatrix}
\begin{bmatrix} \bar{p}_1^i \\ p_2^i \\ \vdots \\ p_{m-1}^i \\ p_m^i \end{bmatrix}.
$$

The complete aircraft queue evolution can now be obtained by simple, and more efficient matrix multiplication. Now p_0^{i+1} and p_1^{i+1} can still be computed by

$$p_0^{i+1} = (p_0^i + p_1^i)e^{-a_i} = \bar{p}_1^i e^{-a_i}$$

and

$$p_1^{i+1} = \bar{p}_1^{i+1} - p_0^{i+1}.$$

Thus far, the method description closely follows Ref. 14. In Koopman's model μ is supposed to equal $\mathrm{E}(\tau)$ for the precise epoch the pair of aircraft is landing in. Clearly, the system of recurrence relations is then actually meant to provide a rough estimate to the non-stationary $M/D/1$ queue. The objective here is to utilize the model's simple structure to provide a quick and robust estimate to the transient Erlang queues. With this in mind, candidate estimates for μ can be

$$\mu = \mathrm{E}(\tau)$$

as in Koopman's model for the $M/E_k/1$ queue or

$$\mu = \frac{\mathrm{Var}(\tau)}{\mathrm{E}(\tau)} \left[\frac{\mathrm{E}(\tau)^2}{\mathrm{Var}(\tau)} \right],$$

for the $M/E_k^{\mu}/1$ queue. Here τ is as appropriate for that period of time the aircraft pair is landing in. Despite the fact that some stationary queuing systems are known to behave quite insensitive to the actual distribution of the service rates and the fact that none of the Erlang models seems to provide a consistently better density estimate, based on intuition the latter definition of μ is preferred here.

As the state probabilities for this queuing system are only evaluated on a sequence of fixed points $t_1, t_2, t_3, \ldots$ in time, it can be argued that such a restricted approach can potentially provide much too coarse delay estimates. However, a refinement in the time intervals can easily be made. Instead of directly computing the state probability shifts over an entire interval step, the time increment can be partitioned into k subintervals, and the calculations can then be performed in finer steps to an adapted set of recurrence relations. Note the clear analogue between the deterministic and the refined deterministic model compared to how the Markov and the Erlang model relate to each other. Despite the resemblance there is now no particular objection to keeping the amount of subintervals fixed, as in the end the service time distribution is not altered. With such an interval refinement theoretically the stack evolution can be determined more accurately. The practical costs of this extended method are exactly k times those of the original model. However, despite all the expenses paid, the results obtained are not significantly different.

Delay metrics

When considering stationary queuing systems, the average waiting time can easily be obtained. As for such systems the Little equation, given by

$$\mathrm{E}L = \bar{\lambda}\mathrm{E}W$$

applies. Here L and W denote the number of customers present and the sojourn times in the system. The parameter $\bar{\lambda}$ depends on the arrival parameters λ_i and the stationary probabilities p_i for all the states i through

$$\bar{\lambda} = \sum_i \lambda_i p_i.$$

Hence the average experienced delay can be determined by a simple inspection of the average queue length. For obvious reason such an argumentation cannot hold for non-stationary conditions, but again the chosen model's simple structure will show to be convenient. Consider the sequence point t_i. The supposed k^{th} aircraft in stack at t_i, assuming a first come first served service process, will not leave the system until sequence point t_{i+k} and its experienced delay is then simply given by $\mu(t_i) + \mu(t_{i+1}) + \cdots + \mu(t_{i+k-1})$. With this in mind, measures as the average remaining delay at time instant t_i or the average amount of work present in the system at time t_i can be computed for every i. Note that, as for this k^{th} aircraft in stack $\mu(t_{i+k})$ accounts for the time needed by the aircraft to actually land this does not form any delay. Here the fact that $\mu(t_{i+k})$ may differ from $\mu(t_i)$ for the unconstrained system and hence does perhaps include some gain or loss is neglected. Also remark that the measure remaining delay, much as the amount of work present in the system can only provide a notion of the actual experienced delay.

6 Model parameters

6.1 Aircraft airside performance parameters

The runway capacity model as stated thus far heavily supports on aircraft performance measures as average times on covered distances and presumed variation to these by means of desirable distributions in terms of model tractability.

6.1.1 Aircraft track measurements

To obtain estimates of the aircraft time variables, and a notion to what extend the distribution assumptions made are justifiable, data out of the NLR developed Flight track and Aircraft Noise Monitoring System (FANOMOS) as in use by the Luchtverkeersleiding Nederland (LVNL) is observed. With this system every air movement, as for instance arriving or departing flights and fly-overs, in the vicinity of Schiphol Airport is being tracked. An example of a FANOMOS measured data sample is given in Table 3.

1836938 Arrival 1999-04-03 06:46:07 07:01:21

0.0	151286.6	476064.2	2432.2	159.6	0.0
4.0	150696.6	476260.8	2358.4	151.9	621.8
8.0	150117.1	476401.2	2293.5	146.7	1218.2
12.0	149545.7	476498.6	2236.4	143.4	1797.8
16.0	148980.1	476565.6	2186.1	141.6	2367.4
20.0	148418.0	476615.3	2141.5	140.7	2931.7
24.0	147857.1	476660.8	2101.7	140.8	3494.4
28.0	147295.2	476715.0	2065.4	141.7	4058.9
32.0	146729.9	476790.2	2031.8	143.5	4629.1
36.0	146160.8	476889.7	2000.2	145.3	5206.9
⋮	⋮	⋮	⋮	⋮	⋮
904.0	108993.2	477145.9	111.2	74.6	93440.6
908.0	109248.1	477303.0	95.7	75.1	93740.0
912.0	109504.9	477460.5	80.3	75.6	94041.3

Table 3 Example of a FANOMOS track data file for an arriving flight.

The information in the header contains a track identity, the flight type, the date, the start and end time of the track measurements. The track data fields include the time offset, the aircraft position, the groundspeed and along track distance. Without the corresponding FANOMOS track flight plan this information is rather useless. In Table 4 an overview of such a flight plan is given. Both files of information can be conveniently linked by the aircraft track identity.

1836934	1999-04-03	06:57:00	06	APP	KLM588	MD11
1836935	1999-04-03	06:59:00	06	nW	KLM642	B743
1836936	1999-04-03	07:00:00	06	dO	CPA271	B744
1836938	1999-04-03	07:02:00	06	dO	KLM440	B763
1836940	1999-04-03	07:03:00	06	APP	SIA328	B744
1836941	1999-04-03	07:05:00	06	dO	MAS16	B772
1836947	1999-04-03	07:18:00	06	dZW	SAB731	A321
1836950	1999-04-03	07:23:00	06	APP	SIA7376	B744
1836952	1999-04-03	07:25:00	06	APP	KLM844	B744

Table 4 Example of a FANOMOS track flight plan for arriving aircraft.

Other fields of interest within the flight plan for instance include the date and time, the airport runway in use and the arrival route, the aircraft call sign and last but certainly not least the ICAO aircraft type designator. With this type designator the ICAO wake vortex classification of the track data generating aircraft can be determined. The data provided in Table 3 belongs to the flight with call sign KLM440. Besides the fact that this aircraft is neatly positioned between a Cathay Pacific and a Singapore Airlines B747-400, it can be concluded that this generating aircraft itself is a B767-300.

6.1.2 Aircraft time parameter estimation

Before obtaining time estimates out of FANOMOS data, some prior assumptions have to be made. As the data is obtained through radar information, there is a time lap of 4 seconds between successive measurements. To overcome this gap, the first assumption is that the speed measurements can be interpolated linearly to obtain the data values for the intermediate time instants. To prevent the need to discriminate on followed approach path or actual location of the glide slope intercept, it is assumed that the measured track distance can be projected along the nominal glide slope path. Thirdly, as the data tracks end as aircraft drop below the radar reach, it is assumed that the aircraft maintain a constant speed from thereon until they reach the runway threshold.

Estimation method

Given these assumptions, the required average times can be computed. The track distance can be formatted such that the result indicates the distance to the runway threshold for the corresponding measurements. This can be done by means of correcting for the remaining distance to the runway

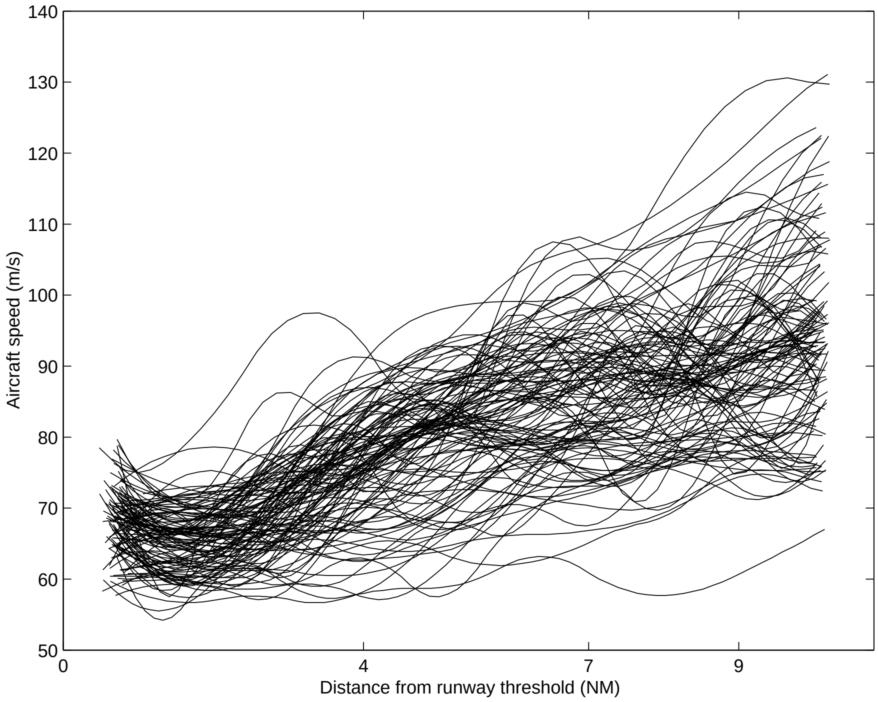

Fig. 5 Approach speeds for an aircraft class consisting of Medium jet like B737 and A320. Measurements consist of a single day of Medium jet arrivals for Schiphol Airport runway 19R as recorded by FANOMOS on April 3rd 1999.

threshold at the final data measurement and for the glide slope angle. With these new track distances one can indicate the two measurement points, between which the beginning of the common approach path S is located. Denote these points with i_1 and i_2. Then let $D(i_k)$, $T(i_k)$ and $V(i_k)$ denote the distance to the runway threshold, the time offset and the velocity for the respective measurement point $k = 1, 2$. Similarly, let i_3 denote that measurement of least distance to the runway threshold that is still not closer to the threshold than the second point of interest s, either being a separation point, the outer marker or the runway threshold itself. Now there are several methods to evaluate the time needed to traverse the distance $[s, S]$. One of these possible methods to derive time estimates consists of two steps and supports on the mentioned aircraft velocity interpolation. As the uncertainty to the actual entry location of an aircraft has an immediate effect on the average speed and its variance, the first step consists of simply taking the difference in time offset between one of the measurement points closest to S, either being i_1 or i_2 and i_3. Here selection between i_1 and i_2 occurs by means of the outcome of a proper binomial distributed random variable. Denote the obtained offset difference with t_1. The second step consists of computing the time needed between i_3 and the point s itself. Let, if it exists, i_4 denote the first measurement closer to the runway threshold than s. If the speed measurement for both measurement points around s is the same, i.e. $V(i_3) = V(i_4)$ this time t_2 is given by

$$t_2 = \frac{D(i_3) - s}{V(i_3)}.$$

Clearly, if i_4 does not exist, i_3 is actually the last measurement. Then the previous statement still stands as the velocity is assumed constant towards the runway threshold. If alternatively, the velocities $V(i_3)$ and $V(i_4)$ differ, t_2 is given by

$$t_2 = \frac{D(i_3) - D(i_4)}{V(i_3) - V(i_4)} \log \left(\frac{V(i_3)}{\frac{s - D(i_4)}{D(i_3) - D(i_4)}[V(i_3) - V(i_4)] + V(i_4)} \right)$$

Now the time estimate for $T_{[s,S]}$ is simply $T_{[s,S]} = t_1 + t_2$. Of course another rough estimate is provided by simply splitting the 4 second inter measurement lap, if applicable, in time intervals proportional to the respective distances of i_3 and i_4 to s.

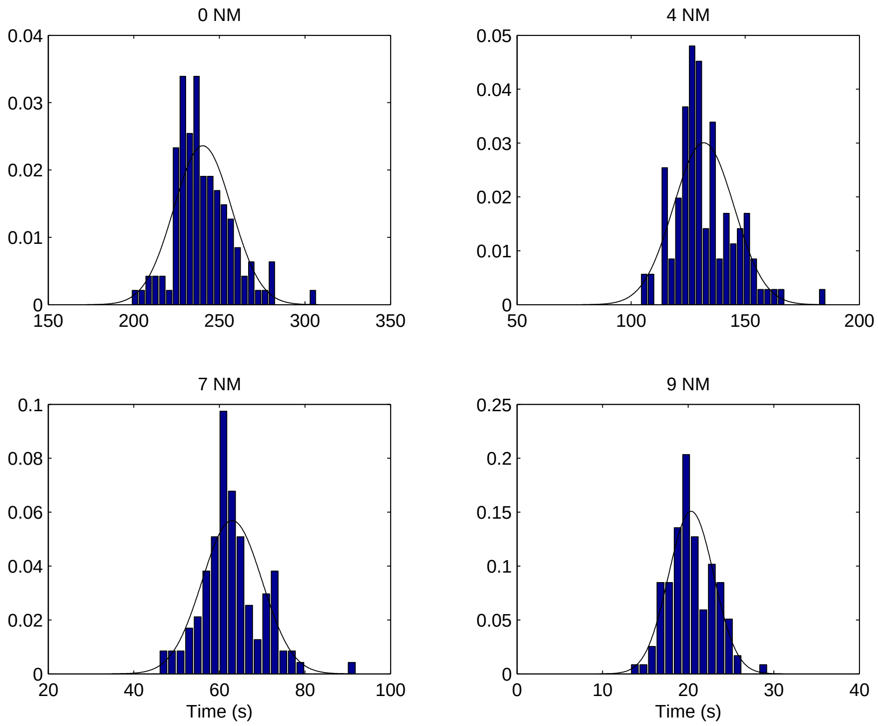

Fig. 6 Time distribution on selected locations for given Medium jet class. Measurements consist of a single day of Medium jet arrivals for Schiphol Airport runway 19R as recorded by FANOMOS on April 3rd 1999. The beginning of the common approach path is chosen at 10 NM off the runway threshold.

Another closely related method to evaluate time parameters consists of a single extra step. The only difference with the previous description is that the time offset is now taken between i_2 and i_3 and the time needed to traverse $[i_2, S]$ is, as that for $[s, i_3]$, evaluated by means of either time or speed interpolation. Both of these described methods have their theoretical advantages, depending on which model representation is used.

Method results

Given a sample of data tracks for an aircraft type or class, estimates for the mean and variance to the average times can be obtained. In Figure 5 a sample of Medium jet arrivals for Schiphol Airport runway 19R is presented. In Figure 6 the results obtained for the very same aircraft sample is depicted for the runway threshold (0NM), the outer marker (4NM) and the 7NM and 9NM points, which are the possible separation points (MEDIUM behind MEDIUM/LIGHT and MEDIUM behind HEAVY) given the conventional separation minima as prescribed by ICAO regulations. Although these results provide no particular evidence towards assumption validity whatsoever, they do provide some assuring feeling.

6.1.3 Aircraft speed profiles

It should be explicitly noted that any on ground measuring system such as FANOMOS, can only gauge the aircraft's groundspeed. Different aircraft speed measures include the indicated air speed (IAS) and the true air speed (TAS). The IAS is the speed as measured within the aircraft. The TAS is the IAS but then corrected for the height at which the aircraft travels. Differences with the IAS are mainly due to the prevailing air pressure. Both of these speed measures can be obtained by inspecting data tracked by the aircraft onboard computer. The groundspeed can be obtained from the TAS by correcting it with the head or tailwind. Hence the FANOMOS measured data tracks thus already contain bias with respect to wind velocities. To obtain capacity measures for different wind conditions one seemingly only needs to provide appropriate data samples. Clearly this is not an optimal situation since there is no control whatsoever on both aircraft and wind velocities and their variances. In order to overcome this problem some effort to obtain aircraft velocity measures out of general aircraft and wind speed profiles was rewarded. The method is illustrated for the aircraft class data sample that is depicted in Figure 5. In Figure 7 the average speed (solid line) and the region spanned by the standard deviation (dotted lines) as functions of the distance to the runway threshold is depicted for both the data sample and an abstract theoretical speed representation (dashed lines). The objective is to use statistics based on the data sample to determine the model's time estimates for the theoretical speed profile. Here some measured information is still needed since inventing some speed profile does not immediately also provide a measure of how the aircraft actually perform within the profile. The method basicly captures this missing information by means of a very simple shooting mechanism. The estimates presented

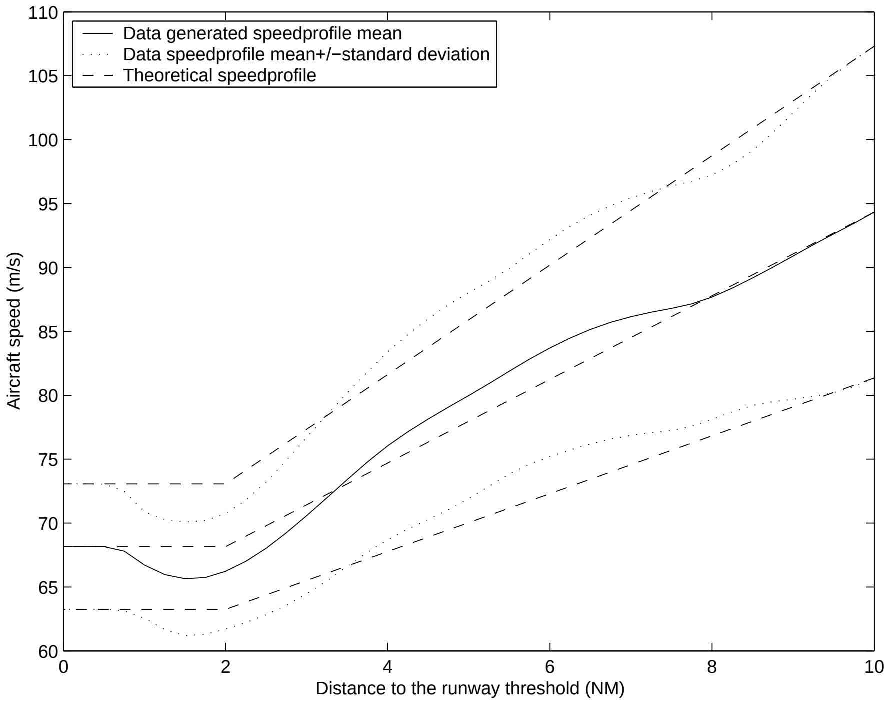

Fig. 7 Speed profiles for an aircraft class consisting of Medium jet like B737 and A320. Measurements consist of a single day of Medium jet arrivals for Schiphol Airport runway 19R as recorded by FANOMOS on April 3rd 1999.

here are based on a single day of measurements and reflect on aircraft average speed parameters, as again those parameters provide a more intuitive notion of the method. Inspection of several significantly different data samples indicates a clear pattern and provides some intuitive proof that the method in itself is sound. However, as long as no extensive validation/rejection study is performed, the method can be considered questionable. On rejecting the method possibilities can perhaps be provided by demanding a more extensive definition of the aircraft speed profiles in terms of the mean, variance and additionally covariance of the speed distribution.

Estimation method

For a data sample the average velocity $V(x)$ and variance $dV(x)$ can be easily computed for a number of fixed locations x along the common approach path and these parameters hence generate a speed profile. With the same effort also the sample's average velocity function V_x and variance dV_x can be computed for some locations x on the approach path by means of the methods described in the previous section by simply correcting the obtained time estimates for the travelled distance. Now V_x and dV_x generate the sample's average speed profile. It should be remarked specifically that V_{x_i} is the average speed as achieved on travelling from the beginning of

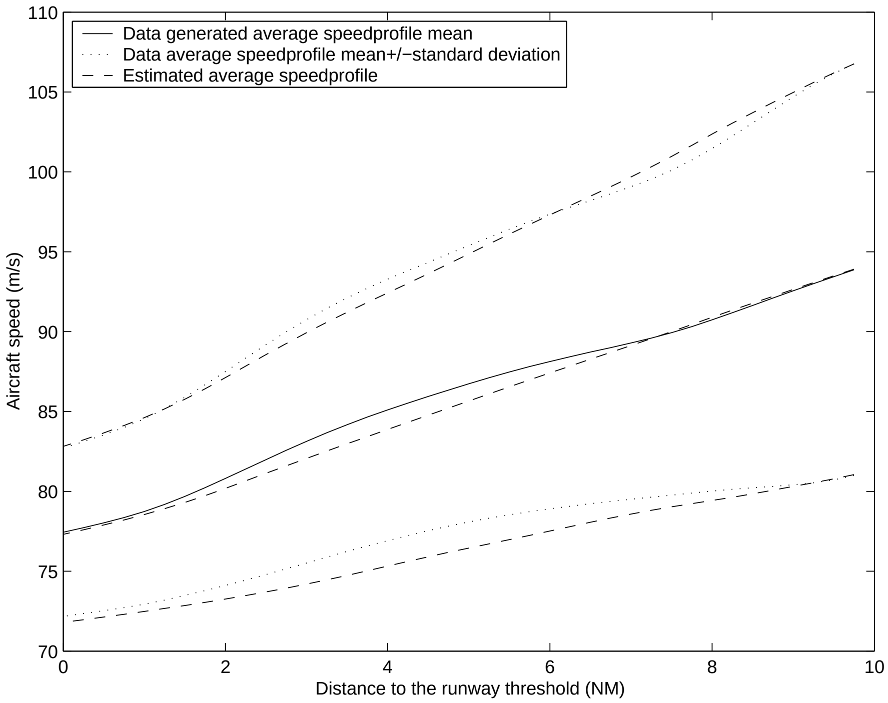

Fig. 8 *Average speeds for an aircraft class consisting of Medium jet like B737 and A320. Measurements consist of a single day of Medium jet arrivals for Schiphol Airport runway 19R as recorded by FANOMOS on April 3rd 1999.*

the common approach path S towards location x_i whereas $V(x_i)$ is merely the average speed on passing position x_i. Note that given this subtle difference $V(x)$ can be any wild function whereas V_x is a far more smoother function as it accounts for some approach speed history. The main question that now arises, is whether a simple relation between $V(x), dV(x)$ and V_x, dV_x holds besides the obvious correspondence as they emanated from the same data sample. In other words, does a relation exists such that V_x, dV_x can be obtained out of $V(x), dV(x)$ by means of some additional knowledge on aircraft behaviour that can be obtained out of an inspection of data samples, but that does not depend on some similarity between the actual track data distributions within the different samples.

Let $V^0(x) = V(x)$, $V^1(x) = V(x) + \sqrt{dV(x)}$ and $V^{-1}(x) = V(x) - \sqrt{dV(x)}$ denote the average speed and the respective average speed plus or minus the standard deviation speed tracks. Then for each of these fixed tracks the average speed functions V_x^0, V_x^1 and V_x^{-1} can be computed through

$$V_x^i = \frac{S - x}{\int_x^S \frac{dy}{V^i(y)}}.$$

Note that again, although initially stated in terms of speed, time estimates can be as readily obtained. Now for any $x_i \in [0, S]$ there exist some constants $A^0_{x_i}$, $A^1_{x_i}$ and $A^{-1}_{x_i}$ such that

$$V^0_{x_i} = V_{x_i} + A^0_{x_i}\sqrt{dV_{x_i}},$$
$$V^1_{x_i} = V_{x_i} + A^1_{x_i}\sqrt{dV_{x_i}}$$

and

$$V^{-1}_{x_i} = V_{x_i} + A^{-1}_{x_i}\sqrt{dV_{x_i}}.$$

Denote with A^0_x, A^1_x and A^{-1}_x the set of all these respective constants on the entire common approach path. Inspection of significantly different data samples, both in terms of sample size and suspected wind conditions, suggest that the A^i_x for $i = 0, 1, -1$ are fairly continuous functions that, more importantly, are rather invariant to shifts in mean and variance of speed profiles generated by different data samples.

Now let $\overline{V}(x)$ and $d\overline{V}(x)$ denote the theoretical speed profiles' mean and variance. If it is assumed that A^i_x for $i = 0, 1, -1$ captures the aircraft speed variability throughout the common approach path, the theoretical average speed profile's mean $\overline{V}_x$ and variance $d\overline{V}_x$ can be estimated by solving

$$\begin{bmatrix} \overline{V}^0_x \\ \overline{V}^1_x \\ \overline{V}^{-1}_x \end{bmatrix} = \begin{bmatrix} 1 & A^0_x \\ 1 & A^1_x \\ 1 & A^{-1}_x \end{bmatrix} \begin{bmatrix} \overline{V}_x \\ \sqrt{d\overline{V}_x} \end{bmatrix}$$

by means of, for instance, a least square estimator. Here the definition of $\overline{V}^i_x$ is similar to that of V^i_x for $i = 0, 1, -1$. The contribution of these estimates $\overline{V}_x$ and $d\overline{V}_x$ to the model is that now at least the aircraft's true airspeed and the experienced wind speeds can be controlled by means of user specified speed profiles that constitute the groundspeed profiles' parameters $\overline{V}(x)$ and $d\overline{V}(x)$.

6.2 Landside performance parameters

The aircraft landside performance parameters mainly reflect on assumptions on the runway occupancy times and their distributions. On evaluating airport runway capacity these parameters constitute the second most important aspect of the model, since the runway occupancy based maximum capacity will be the dominant factor in determining an airport runway capacity upper bound. Recall that, as briefly mentioned in section 2, besides the runway layout also the ambient weather circumstances affect the duration of runway occupancy.

6.2.1 Runway occupancy times measurements

With regards to runway occupancy, it is not clear whether an automated system as FANOMOS for airside performance is being used on a regular basis at Schiphol Airport. The measurements used for this study where obtained at Schiphol Airport, by means of a digital stopwatch. The observers collected data from a central location, the control tower, which provides a clear view of both the runway threshold and exits. Because of the long distance from the observation point and the runway threshold and exits, an accuracy level of ± 4 seconds was annotated. These ROT data measurements were for the purposes of this study broken down by runway in use and aircraft class. Unfortunately, the records did not include an indication on prevailing weather conditions during the measurements.

6.2.2 Runway occupancy times simulation

As the runway occupancy times can significantly differ given the weather conditions, a simple simulation model was developed and results, for a given baseline parameter scenario, where compared to those for obtained ROT measurements.

Estimation method

In order to obtain the ROT distribution by means of a simulation for different weather circumstances as a result of the runway layout, some simplifying assumptions are made. It is assumed that the position the aircraft touches down, the aircraft's brake way and also the exit speed, according to the prescribed exit type, follow uniform distributions. The aircraft's velocity over the threshold is supposed to follow a normal distribution, its speed remains constant until the touch down point and from thereon the aircraft uniformly decelerates until it reaches the exit speed at the turn off. Furthermore, the aircraft utilizes the first possible runway exit it encounters that is allowed in terms of its brake way. If the touch down point is given by T_{DP}, the distance to the exit by D_e, approach speed V_a and exit speed V_e and the co-ordinate origin is at the threshold, the ROT is then simply given by

$$ROT = \frac{T_{DP}}{V_a} + 2\frac{D_e - T_{DP}}{V_a + V_e}.$$

Within this simple model, characteristics as location of the aimed touch down point and the locations of the runway exits can be chosen such that they represent the actual airport's runway layout. The aircraft brake way, approach and exit speed may depend on the aircraft type and weather conditions. However factors that do not immediately influence the identified variables, like the amount of crosswind or the actual location of the runway relative to the airport terminal, cannot be easily incorporated in the model, whereas they do have influence on the actual roll out times.

Method results

Part of the model's success can be explained by the fact that, in not specifying a very detailed deceleration process a large number of different arrival combinations in terms of both aircraft speeds and exits used are evaluated. This then reasonably accounts for the enormous diversity in aircraft times spend on the runway, as for example a pilot's choice may sometimes be between just missing a runway exit and hence being forced to taxi towards the next one, or risking the loss of frequent flyers while winding off the runway too enthusiastically. In Figure 9 the normal approximation to the simulation results for a fixed baseline parameter choice, based on Medium jet aircraft performance, is compared to the actual measurement data of the study described previously. It should be mentioned that, in terms of accuracy, a different underlying distribution estimate, would better describe the situation. Broken down to runway exits a normal density can very well represent the runway occupancy times obtained through simulation. For multiple exits a mixture of normal densities would then seem more appropriate. However, such an estimate choice would eliminate the capacity model's simple structure.

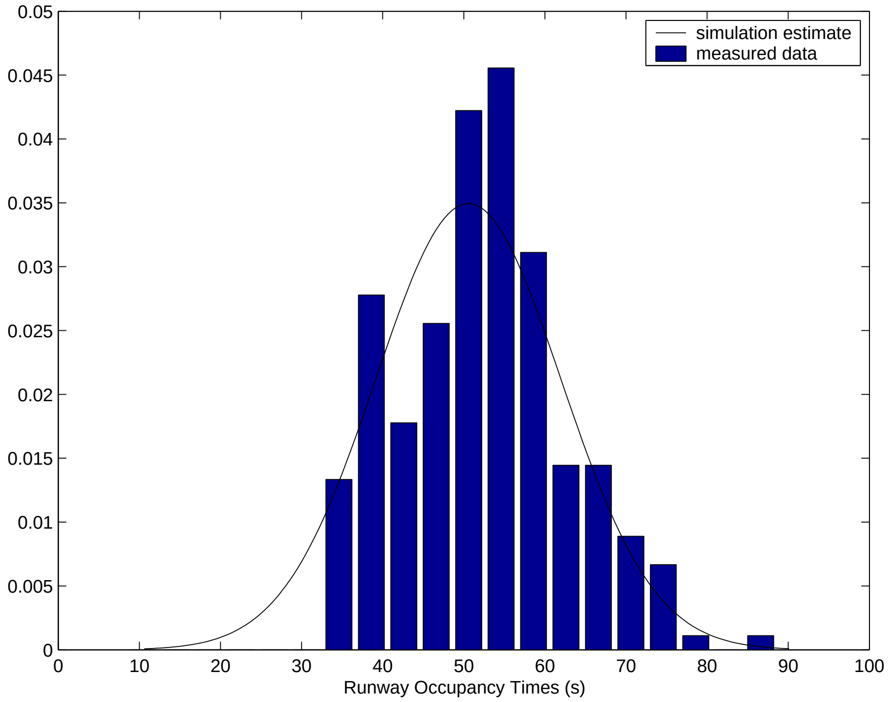

Fig. 9 Runway occupancy times distribution and simulation estimate for a Medium jet class. Measurements consist of several days of Medium jet arrivals for Schiphol Airport runway 19R.

Method validity

On a first inspection, the proposed method consists of a neat first order description of the aircraft roll out process and seemingly provides a means of evaluating the shifts in both average and variance of the runway occupancy times distribution, based on the different proportions of aircraft that use a certain runway exit given some weather circumstances. It should be emphasized that the model is based on very simple assumptions of which it is questionable whether they actually hold in practice and that, in the absence of a detailed runway occupancy study, some caution should be taken when using the obtained simulation results.

7 Model results and validation

Usually validation of a model requires a thorough examination of the actual performance in regards to the situation it ought to describe. As resources in terms of time available for this study were limited, a careful sensitivity study is not yet performed. In the next sections some examples of the numerous theoretical possibilities of the proposed model are given and some additional notes on these results are given to indicate their sense of validity.

7.1 Parameter estimation

Model parameter estimation is restricted to a single day of measurements and a fixed runway in order to isolate, as much as possible, the actual aircraft performance. Although weather, and indeed more importantly the prevailing wind conditions generally tend to vary throughout the day and hence continuously perturbate the model variable estimates, this restrictive approach yields less wind polluted parameter estimates than those obtained when considering larger samples of multiple days or runways. The major drawback of this estimation method is that, as the sample sizes for the different aircraft classes are according to the actual traffic mix for the particular day of measurements, the accuracy of the estimates may vary for the different aircraft classes.

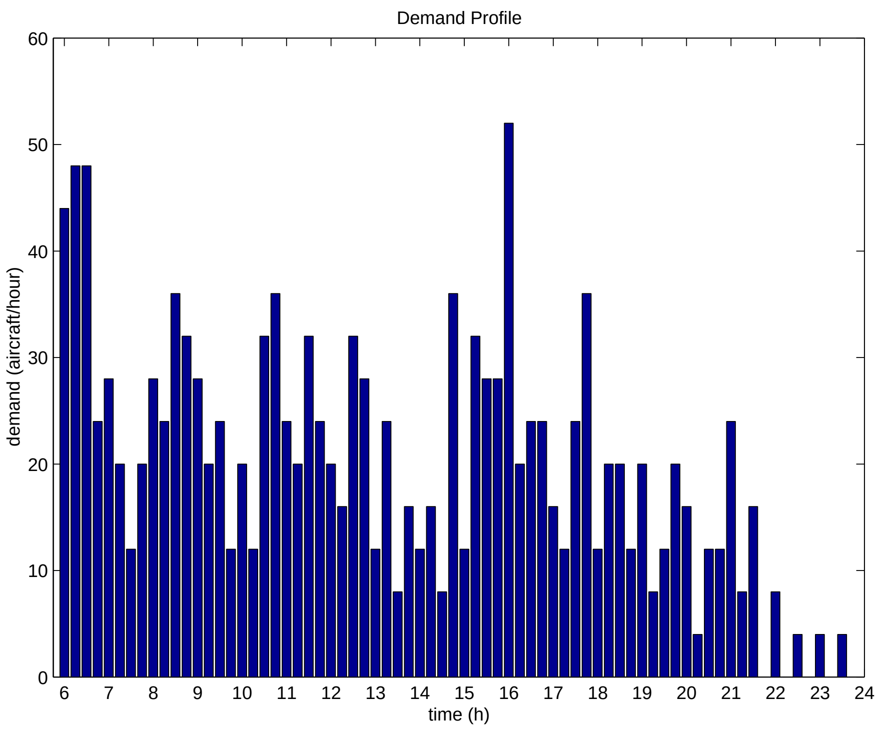

Fig. 10 Fictitious demand profile for a single runway.

7.1.1 Aircraft classification

The aircraft types are not only distinguished by the rather coarse ICAO wake turbulence classification, but additionally by a more delicate categorization as used in the NLR research programme. The aircraft class categorization consists of Large jumbo jet like B747, Wide body jet like A300 and B767, Medium jet like A320 and B737, Regional jet like F70/F100 and DC9 and Medium turbo prop like F27/F50 and ATR42. This classification is based on both wake generating and approach speed characteristics. For completeness also the Light turbo prop like Beech99 should be mentioned, but these are discarded for further use as they form a negligible part of the daily traffic mix at Schiphol Airport.

7.2 Capacity model

As an example some results based on airside parameters obtained through analysis of every single aircraft arrival on Schiphol Airport runway 19R on April 3rd 1999 are presented. Instead of providing a complete sensitivity study for the variation in traffic mix, more concise results are provided for the hourly traffic mix variation experienced during the day. For some precautionary reasons the runway occupancy times are as well obtained by data measurements for Schiphol Airport runway 19R, as the runway occupancy model performs precarious. The runway occupancy

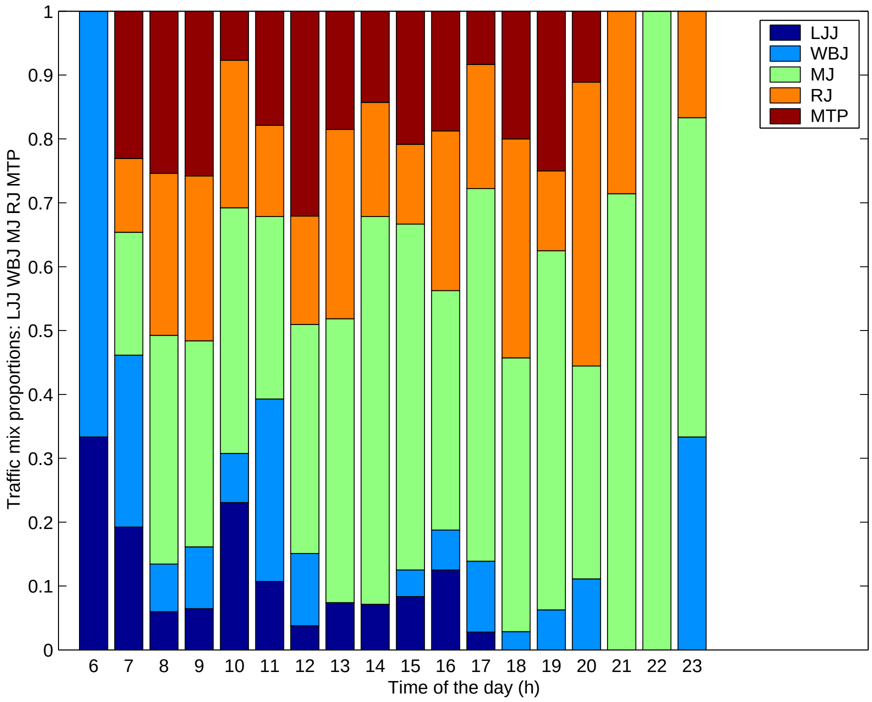

Fig. 11 Hourly traffic mix as obtained by analysis of the FANOMOS flight plan for Schiphol Airport on April 3^{rd} 1999.

time estimates are based on measurements for a fixed common runway exit for each respective aircraft class.

7.2.1 Distance separation constraints

The most obvious restriction to airport runway capacity is the required distance separation constraint as prescribed by safety regulations. As the objective of this study is to develop a runway capacity model capable of obtaining capacity estimates for different aircraft spacing scenarios this paper cannot be considered complete without an actual example for different distance matrices. In Figure 12 results are depicted for the traffic mix proportions provided in Figure 11. The used distance scenarios are the ICAO separation matrix, as defined in Table 1, as baseline and two imaginary matrices being the ICAO constraints reduced by 0.5 NM and 1.0 NM, respectively. For the fourth scenario depicted, no distance constraint is required and separation occurs only on the basis of the runway occupancy constraint. Note that the capacity increase is indeed not linearly proportional to the decrease in separation distance, but that is not necessarily due to the aircraft deceleration, as on applying reduced distance separation constraints eventually also more aircraft

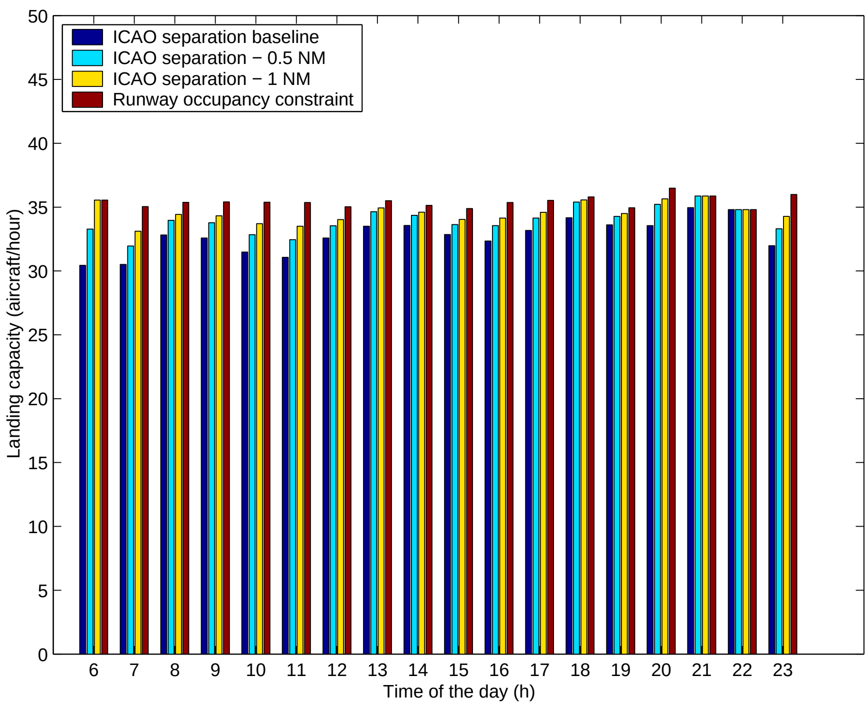

Fig. 12 Capacity sensitivity to actual separation distance. Estimates are obtained for the ICAO separation matrix, simple variants to these distances and the runway occupancy constraint. Aircraft are spaced longitudinally at an outer marker located 4NM off the runway threshold.

combinations are restricted by the runway occupancy requirement.

7.2.2 Wind conditions

Another important factor directly influencing the airport runway capacity is the actual wind speed as experienced by landing aircraft. Given an amount of head wind, the aircraft ground speed drops with the experienced wind strength. This speed decrease generally degrades the runway capacity as now more time is needed to cover the required separation distance. In Figure 13 some capacity estimates are provided for the corresponding traffic mix proportions as depicted in Figure 11. Similarly, one can imagine that the actual variation in wind speed can influence the runway capacity as aircraft speed then tends to vary to a greater extend.

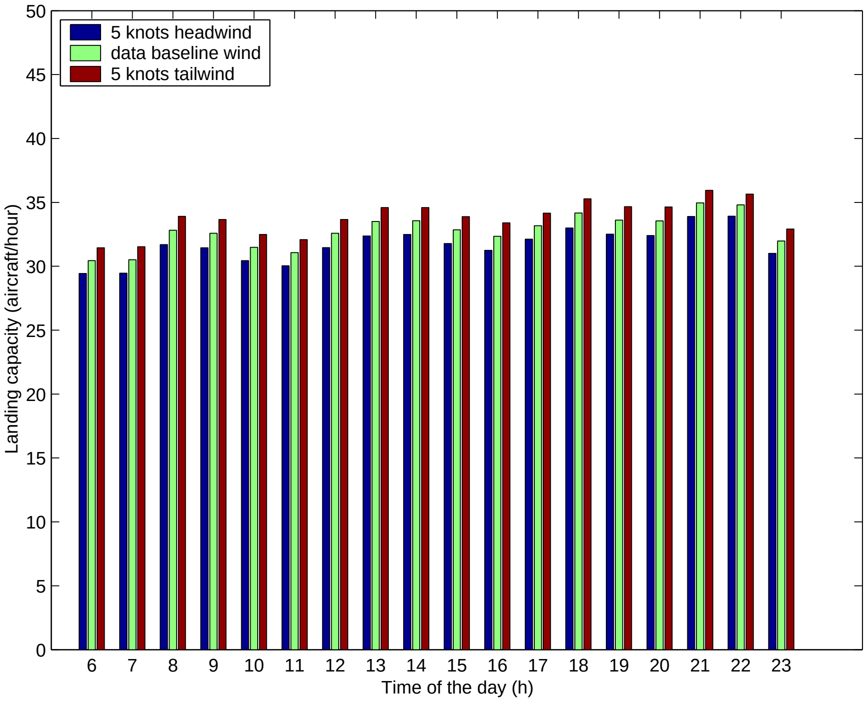

Fig. 13 Capacity sensitivity to head or tailwind. The measured parameters are additionally perturbated by either 5 knots head or tailwind to obtain these estimates. Aircraft are spaced longitudinally at an outer marker located 4NM off the runway threshold according to the ICAO separation matrix.

7.3 Delay model

Delay results can be obtained by combining the runway capacity estimates of the proposed capacity model and a realistic demand profile for an airport operating on a single runway arrival mode. The example provided here is the direct application of the results as provided by the capacity model example for the ICAO distance separation constraints and no additional wind perturbations. The

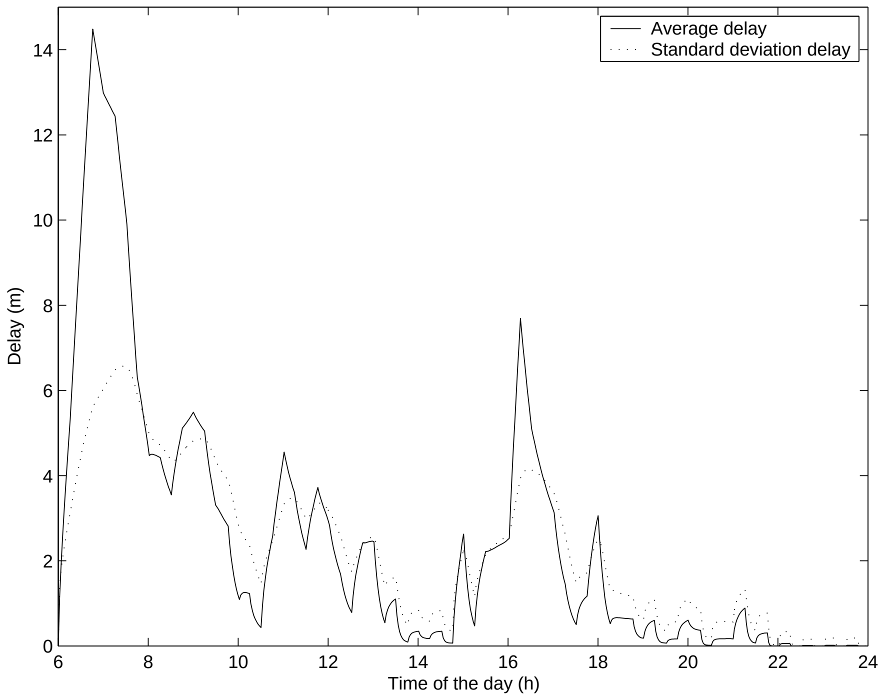

Fig. 14 Average remaining delay for given traffic and demand and runway capacity profiles. Runway capacity is determined on the basis of the ICAO separation matrix applied at an outer marker located 4NM off the runway threshold.

traffic demand profile is partitioned to expected numbers of arrivals per quarter of an hour and is depicted in Figure 10. Results for the average remaining delay are provided in Figure 14. In Figure 15 the reduction in average remaining delay is depicted for the case that the ICAO distance separations are each decreased by 0.5 NM. While a flight reduction of for instance 30 seconds may not seem particularly much, and will probably not be noticed by any on board passengers, both the eventual economical and environmental impact can be huge.

7.4 Model validity

As the model approach in this paper is not radically new but just a mere step in another direction, some remarks can be made on the model validity.

7.4.1 Capacity model

Thus far, when presenting the proposed runway capacity model with parameter estimates based on a single day of aircraft arrivals for a fixed runway at Schiphol Airport for the ICAO distance separation matrix, realistic results compared to actual performance at airfields were obtained. It can thus cautiously be assumed that modifying both the aircraft motion and the ATC spacing

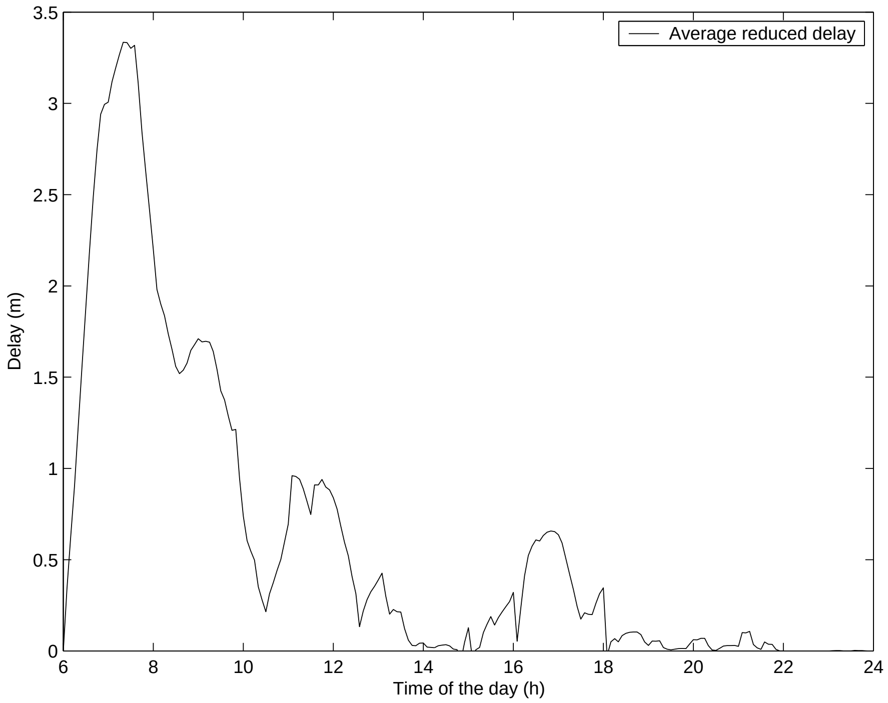

Fig. 15 Average reduced remaining delay for given traffic demand and runway capacity profiles. Runway capacity is determined on the basis of the ICAO separation matrix and the ICAO separation matrix reduced by 0.5 NM, applying at an outer marker located 4NM off the runway threshold.

method in the existing LMI capacity model did not alter the model's performance. However, conformable model results compared to actual daily practice can only provide an eye wink towards model validity.

7.4.2 Delay model

In view of earlier work, both in terms of airport delay modelling and general queuing systems, this paper's approach involved no particular originality or genius. The delay module of the FAA Airfield Capacity Model consists of determining estimates based on both the standard $M(t)/D(t)/1$ and the $M(t)/M(t)/1$ queuing systems. A study on the applicability of queuing models to determine airport delay estimates, specifically for Schiphol Airport, based on the FAA Airfield Capacity Model, was recently performed by Uittenbogaard (Ref. [27]). Within the NASA Aviation System Analysis Capability airport capacity model delay is currently estimated by means of calculating state probabilities for the $M(t)/E_k(t)/1$ queuing model. This approach is extensively tested within the NASA Terminal Area Productivity program.

8 Model extensions

The runway capacity model, as proposed in this paper, only discusses the case of a single runway, operating to capacity, for arrivals only. As the single runway is the main building block of a runway configuration and it is only during saturation conditions that capacity limitations are a problem, these restriction can be justified. In the next sections some model extensions beyond this current paradigm are provided and variants to the proposed methods are discussed. For some of the possible capacity model extensions the delay analogues are presented in the relevant references.

8.1 Runway mode of operations

In this paper little attention is given to the departures only or the mixed operations mode, when arrivals and departures are simultaneously accommodated on the same runway. For departures only an interesting runway capacity model example is provided in Ref. 16. A model describing the full airport departure process of which some further study can be highly recommended is provided in Ref. 10. The problem when considering a single runway that operates in mixed mode is that in modelling the situation, some priority strategy between arrivals and departures needs to be assumed. Examples of such strategies can be strict alternation of arrivals and departures or, for instance, full priority awarded to arriving aircraft, as in theory a departing flight can be kept in hold for an infinite period of time whereas, for obvious reasons arriving aircraft cannot. In the latter example an aircraft departure can only take place if it does not disrupt consecutive arrivals and all the applicable safety regulations are followed. Given such a priority strategy, results of both arrivals and departures only models can be combined to yield mixed operations capacity estimates. A queuing system to obtain delay estimates for mixed operations that involves a multiple queue is provided in Ref. 14.

8.2 Runway configurations

The capacity of a complete runway configuration in use may depend on the capacity of the single components it consists of. For instance, for runways that are allowed to operate independently, capacity is simply the sum of the components' capacities. However not all configurations allow this simple approach. For dependent runway configurations such as closely spaced parallel, converging or crossing runways, operations occurring on one of the runways influence those on the other runways in the configuration. In Ref. 16 an example of how to theoretically accommodate staggered departures on closely spaced parallel runways is given. This method can be directly applied to staggered arrivals, and under similar considerations, extensions towards converging and crossing runways can be developed. The main problem with such extensions is that they provide an estimate to the capacity that can optimally be achieved. In reality an airport rarely operates to runway capacity in regards to the theoretical capabilities of a full runway configuration in use.

A simple reason is that, while runway capacity does provide the main bottleneck during arrival and departure peaks, for instance factors as the airport amount of apron space or even the airfield luggage handling capabilities are also limited. Moreover, the configuration in use will likely alter in the course of the day since it is adapted to prevailing weather conditions and temporary shifts in runway usage demand as the arrival and departure peaks generally do not coincide.

8.3 Controller precision

In the presented model it is implicitly assumed that the controller can always impose the desired fixed time separation $\delta_t + \epsilon_t$ at the entry on the common approach path S. As practice shows that this may not always be the case, one can consider the variant to the model in which ϵ_t denotes a random variable, that is independent of the other stochastic variables, to account for manoeuvring or feeder errors by the ATC. For more general densities of ϵ_t, the distribution function for the inter-arrival time t_{lf} for a fixed leading and following aircraft pair l and f may no longer be normal, but consists of a convolution of a normal random variable and a term for the respective distribution of ϵ_t. Depending on this latter distribution, the density of t_{lf} can then be evaluated. As the effect of ϵ_t on the mean and variance of the inter-arrival time distribution t_{lf} is rather straightforward, depending on the accuracy of the normal approximation, it can be sufficient to state the mean and variance of ϵ_t. For the example estimates provided in section 7, ϵ_t was fixed to $\epsilon_t = 10$ seconds.

9 Conclusion and recommendations

9.1 Conclusion

This document describes the development of an airport runway capacity model capable of investigating the direct benefits in the application of dynamic separation distance requirements. The effort resulted in the further development and implementation of existing analytical runway capacity models. The current model consists of separate modules to derive the relevant modelling parameters, to obtain capacity estimates for a single runway and to provide corresponding efficiency measures. Model parameters, such as inter-arrival times, are evaluated given actual flight track data or speed profiles and any given spacing scenario. The runway capacity model represents aircraft arrival operations during final approach and roll out as allowed by air regulations. Airport efficiency figures are based on delay evaluation for the obtained runway capacity and given traffic demand profiles.

In view of earlier modelling efforts, the runway capacity model suggested in this paper extends the work previously done to the point that now both aircraft speed and motion can be modelled up to an arbitrary level of detail. Additionally, air traffic procedures can be accounted for as performed in actual practice. As a consequence, runway capacity can be evaluated for any given separation distance scheme and the developed model provides an opportunity to accurately estimate the benefits of proposed dynamic, such as weather dependent, separation distance schemes.

9.2 Recommendations

Before using the described model to evaluate the benefits of proposed changes in separation distance schemes, it is recommended that first the model is thoroughly tested and validated under the current air traffic procedures. This can be done by using all kinds of information that is currently available in the whole air traffic management process and hence evaluate the identified capacity and delay metrics either directly or through harmonisation using automated systems. Furthermore, it should be recognised that weather and wind conditions not only influence wake vortex position and strength, but also influence the aircrafts performance and runway occupancy times. These may have an opposite effect on capacity. In particular the effects on runway occupancy time need further investigation, as the runway occupancy requirement eventually determines the runway capacity envelope.

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Appendices

A Model implementation

The model is straightforwardly implemented in several MATLAB functions. To provide users easy access, the MATLAB functions are integrated in tools, using SPINEware, that together constitute a workflow. For practical purposes, a brief description of the main functions within these blocks is provided in this section.

A.1 Parameter estimation

The model parameters can be based on actual flight track measurements or user specified aircraft speed profiles.

A.1.1 Aircraft track measurements

Data files of aircraft track measurements, for instance obtained through FANOMOS, serve as primary input for the tool called *track_data.m*. Further input parameters consist of the location of the beginning of the common approach path, and the actual runway to monitor. Given these parameters, unused content is removed from the data files and the track measurements are classified according to the categorization sketched in section 7.1.1. The ICAO classification can be obtained by a simple adjustment. In the function called *time_data.m* the related data tracks are analyzed for the mean and variance of both the time estimates for a series of locations on the common approach path and the aircraft speed over the runway threshold. This function also derives the information needed when considering aircraft speed profiles.

A.1.2 Aircraft speed profiles

User specified aircraft speed profiles and wind scenarios serve as input for the tool called *time_est.m*. Its role is somewhat similar to that of *time_data.m*, but now perhaps for multiple aircraft ground speed scenarios. An additional option included is the use of FANOMOS based speed profiles. This provides an opportunity to additionally perturbate these track measurements.

A.1.3 Model time estimates

In the tool called *time_model.m* results for either the *time_data.m* or the *time_est.m* functions are converted to the actual model aircraft time parameter estimates. In order to achieve this, the location of the fix where the aircraft are to be longitudinally spaced, for instance the runway threshold or the outer marker, and the applicable distance separation standards should be specified as input. Multiple separation matrices for different spacing conditions can be evaluated. The tool called *land_sim.m* derives the runway occupancy time estimates given a runway layout. The aircraft approach speed distributions used here, are obtained either in *time_data.m* or *time_est.m*.

A.2 Capacity and delay estimation

The capacity and delay modules largely rely on the same infrastructure. In the tool called *IAT_est.m* the inter-arrival times mean and variance are evaluated for every possible combination in terms of aircraft sequencing, speed scenarios, distance separations and runway occupancy distributions. Given profiles of the traffic mix proportions, the actual ground speed, distance separation and runway occupancy scenarios, capacity measures, based on the *IAT_est.m* results, are obained in *capacity.m*. For delay estimates the *delay.m* tool additionally requires the specification of an aircraft arrival demand profile.