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Do variable session lengths per surgical department contribute to a better OR utilization?

Master thesis

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Preface

This Master thesis has been written for my Master Mathematics at Leiden University, under supervision of dr. Floske M. Spieksma. The research has been conducted at VUmc in Amsterdam during an internship of seven months, and was commissioned by drs. ir. Ilse Matthijssen, and Lizan E. Aalders MScBA. My supervisor at VUmc was dr. Gabriella Balke - Budai. Also, prof. dr. Ger Koole was a great help during my internship, without him I possibly would not have finished my thesis at all.

Operation room complex of a hospital
Each hospital with an operation room (OR) facility is trying to optimize their surgery schedule to be able to conduct as many surgeries as possible. Hence, much research has been done into the optimal way of scheduling surgeries: surgeries should be scheduled in increasing order of average surgery duration; surgeries with a high unpredictability duration should not be scheduled in the beginning of the day; and one long session is much better than multiple short ones. Nevertheless, little research can be found about the optimal session length. A session is the time in which surgeries are allowed to be scheduled. Most hospitals work with a regular session somewhere between 08h00 to 15h45 (with some additional extra long session length assigned to some departments). This session length of circa 465 minutes is inline with a typical working day of the hospital staff. However, the different surgery durations that occur at a surgical departments are not always compatible with this length. Meaning, undertime or overtime might occur, giving a suboptimal OR-utilization. So, what is the best session length per surgical department, and do these variable session lengths contribute to a better OR-utilization?

Maximize OR-utilization while minimizing the overtime
Suppose that each surgical department has three groups of surgeries, short ones, surgeries of medium long length, and surgeries of long duration. Each different combination of surgeries might cause some overtime or undertime. We want to know for what session length the expected overtime, and expected undertime will be minimal. Therefore, a mathematical model has been designed to determine the expected overtime and undertime for different session lengths. The model is performed on the historical elective surgeries of 2015. Also, given these expectations, the expected utilization is determined. The model determines the best session length for each surgical department.

Results
The list of expected overtime, undertimes, and utilizations show that for multiple departments an improvement in utilization can be generated by chancing the session length. Some departments need a shorter regular session length, others a longer one. Additionally, it is best to give some departments not just one session length, but two different ones. Fortunately, the determined best session lengths of each surgical department were not all completely different. Therefore, it is rational to implement the best session lengths.

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1 Introduction

1.1 About VUmc

This research has taken place in the form of an internship at VU Medical Center Amsterdam (VUmc), the academic hospital of 'de Vrije Universiteit' of Amsterdam. VUmc is one of eight university hospitals in The Netherlands, and was originally founded in 1964. Since 2001 it is called VUmc, due to the fusion of the hospital and the medical faculty [1]. Over the years, the incentive of VUmc has become 'with renewal contributing to the medical knowledge of tomorrow'. VUmc is a growing hospital with as her core tasks, patient care, education, training and scientific research. It has nineteen operating rooms (ORs), including two new hybrid ORs. VUmc is the first hospital to have these ORs with the newest medical image-guided intervention technology of Philips [2]. Next to this, there are more than 700 beds and over 50,000 patients. The multitude of factors makes the scheduling of surgeries a complicated process.

In 2005 the mission of VUmc was formulated to be distinctive patient care, excellent academic education, and innovative and groundbreaking research. VUmc was founded on the basis of the Christian beliefs, therefore it connects this belief in a personal, honest and respectful treatment of people [3]. Also, ambition and fundamental curiosity are important to VUmc. The vision that VUmc wants to carry out is to contribute to the quality of health care, and thereby to the well-being of people. They also want to be known to be an innovative organization, in which everyone enjoys what they are doing. Generally, ambition, involvement, and carefulness are the three main values of the hospital.

During the internship, the supervisors at VUmc were Lizan Aalders MScBA (Head OR department), drs. ir. Ilse Matthijssen (Manager Strategy and Innovation), and dr. Gabriella Balke-Budai (Staff advisor logistics). From Leiden University, the research was supervised by dr. Floske Spieksma.

1.2 Introduction to the research

The research focuses on the operating rooms (ORs) of VUmc (see Section 1.3.1). A hospital obviously wants to do as many surgeries in a year as possible. Therefore, they have the goal to optimize the OR planning, so more surgeries can be done on the same OR capacity. Much research have been done on the optimization of OR planning. However, our research focuses in another direction. Most of the research done is about the best way to schedule surgeries on an OR session. We have looked at what the best session length is wherein surgeries can be scheduled per surgical department. However, optimizing the surgery schedule is not the scope of this research.

In the rest of this chapter we have given definitions and notation needed to understand this thesis. Followed by the purpose of this research.

1.3 Thesis outline

This thesis is structured as follows. First, we have explained the OR facility, and given important definitions in this research. The next chapter gives the research objective and research questions, as well as some additional information needed to start this research. Chapter 4 explains when a session is called optimal, and Chapter 3 gives a thorough analysis of the realized surgery data of 2015. After that, we explain the mathematical model that we have designed, and the simulations we want to conduct. The results from these simulations are given in Chapter 7, which also states the answer to research question 1. The other research questions are answered in Chapters 8 and 9. Last, we have answered the research objective, and give our vision on future work.

1.3.1 The OR facility

The OR facility of VUmc has nineteen operating rooms, divided over two floors. On the sixth floor, rooms 1 to 10 can be found, and on the second floor rooms 11 to 16. Since 2017 there are three extra rooms available, two hybrid rooms (operating room with medical imaging devices available) and an endo suit (OR specially for minimal invasive surgery). These are rooms 20, 21 and 22, and are also stationed on the sixth floor. Of those nineteen ORs, fifteen are available for elective surgeries. One of the rooms on the sixth floor is always unoccupied and available for emergencies, and it is prefered that one extra room is always free for unexpected circumstances such as more emergency patients or malfunctions in other ORs. In 2017 a 'flex room' has been introduced. Surgical departments can apply for some additional OR-time, which can be awarded when the flex room is still available.

1.3.2 Surgical departments

The table below lists the different *surgical departments* of VUmc. The first seven departments are usually assigned to a room on the sixth floor, the last six to the second floor. Department Gender gets rooms assigned on both floors. Of course, in case of an emergency a department can always move to a different floor. The abbreviations belong to the Dutch translation, these will be used to prevent confusion. [19]

CCH:	Cardiac surgery
LON:	Lung diseases
ORT:	Orthopedics
HLK:	Surgery
NCH:	Neurosurgery
GYN:	Gynecology
URO:	Urology
Gender:	Gender dysphoria
MKA:	Mount diseases, jaw and facial surgery
PCH:	Plastic surgery
KNO:	Ear, nose, throat
OOG:	Ophthalmology
KCH:	Pediatric surgery
URK:	Pediatric urology

Table 1: The surgical departments of VUmc (with Dutch abbreviations)

1.4 Definitions

Elective patients:	Patients known, who can be planned in advance
Non-elective patients:	Patients who are not known when the existing scheduled was made, so they couldn't be scheduled in advance. Also called, *emergency patients*.
Blueprint/ Session distribution:	Schedule of the assigned OR-time each surgical department gets throughout a year, specified per day. *Further explanation in Section 2.5.1*
Surgical department:	A department in the hospital that who performs surgeries.
Surgical case:	The whole surgery of a patient. A surgical case consists of multiple operations.
Surgery number:	Every surgery conducted gets a unique number.
Operations:	A particular part of a surgery on a patient
Operation code:	Every type of operation has an unique code containing six numbers, and sometimes an additional letter at the end.
Patient number:	Every patient gets his/her own personal number when visiting the hospital for the first time.
Session length: (OR-day)	A workday on an OR. In this time, surgical cases can be planned. The current session lengths are: • Late start 1: 09.00-15.45 (405 minutes) • Late start 2: 09.30-15.45 (375 minutes) • Regular: 08.00-15.45 (465 minutes) • Long session 1: 08.00-17.30 (570 minutes) • Long session 2: 08.00-17.45 (585 minutes) • Extra long session: 08.00-19.45 (705 minutes) *Further explanation in Section 2.5.2.*
Session	With this we mean one session length (whatever length) of a department. Scheduled on one OR.
Start OR-time slot:	A predetermined time at which the first patient is scheduled for that OR. *Note:* Usually one of the starting times given above.
End OR-time slot:	A predetermined time at which the OR should close. Optimally, this is the time the last patient leaves the OR. *Note:* Usually one of the closing times given above.
Surgery duration: (Surgery time)	The time it takes to finish a surgical case. I.e., the time between a patient arriving at an OR, and leaving the OR. [20]
Start case:	The time a surgical case actually started. *Note:* Not necessarily the time the cases was planned to start.
End case:	The time a surgical case actually ended. *Note:* Not necessarily the time the case was planned to end.
Over-utilized OR time: (**Overtime**)	Time after 'end OR-time slot', or after the planned end of a case.
Underutilized OR time: (**Undertime**)	Time that is left between finishing the last patient and 'end OR-time slot'
Turnover time:	The time between two patients: The time between one patient leaving the OR and the other entering the OR. We choose it to be a fixed constant of 20 minutes.
Early start:	The start of a surgical case before the actual 'start OR-time slot'
First case tardiness : (Late start)	The difference between the 'start OR-time slot' and the actual 'start case' [14]
Non-operative time:	The sum of 'undertime', 'turnover time', and 'first case tardiness'. [14]
Inside operating time : (IOT)	The time between the start of an assigned session, and the end of that assigned session.
Outside operating time: (OOT)	The time before the start of an assigned session (up to 30 minutes is allowed), and the time after the end of the assigned session when 'start case' was after 'end OR-time slot' (otherwise we call it overtime.)

Below we have given two figures to visually explain some of the concepts given above, to get a better understanding of the concepts we use in this thesis. The first figure shows the specifications of an OR-day, and the second one shows how IOT, OOT, overtime and undertime are defined.

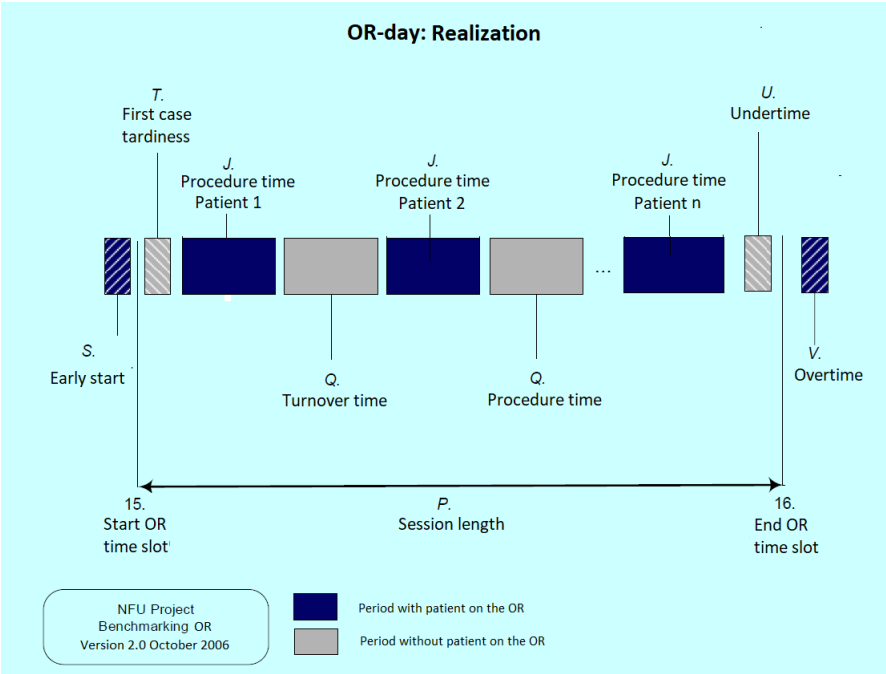


Figure 1: OR-day: realization [20]

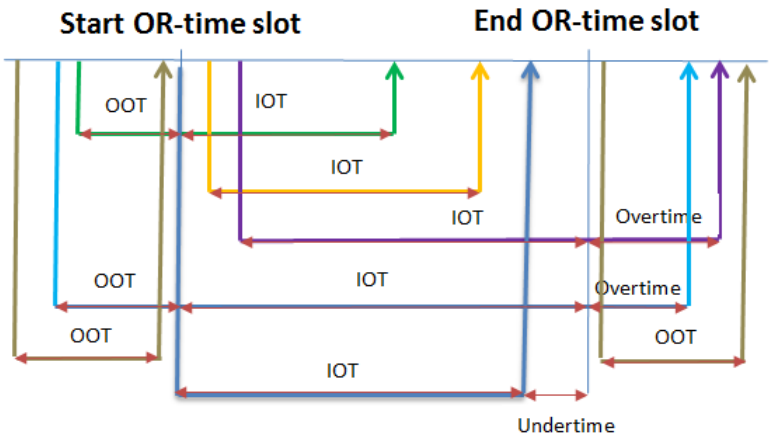


Figure 2: Inside, outside operating time, and overtime

In this research we have only looked at the end of the OR-day. So, we have considered overtime and undertime, but will not take into account an early start or first case tardiness. Technically speaking, first case tardiness is also undertime, because OR-time is left unused. An early start might ensure no or less overtime. Overtime and undertime at the end of the OR-day will likely occur due to bad planning or an unsuited session length.

1.5 List of mathematical notations

Below we have given a list of notations used in the mathematical language in this thesis.

Notations	Meaning
C	Surgical department (Section 1.3.2)
S	Session (length)
S_C	Session of department C
S_A	Amount of sessions allocated
S_R	Amount of sessions used
$S_{m,C}$	The set of all sessions of department C in one year, with $m = 1, ..., j$
d	Surgery duration
$d_{n,m}$	The set of all surgery durations on session $S_{m,C}$, with $n = 1, ..., i$
t_i	Turnover time of surgery i We take $t_i = 20$ minutes $\forall\ i$. Further called t
k	The number of surgery duration groups, with $g = 1, ..., k$
$Q = \{q_1, q_2, ..., q_{k-1}\}$	The boundaries between the groups, with $q_1 < q_2 < .. < q_{k-1}$
$V = \{V_1, V_2, ..., V_k\}$	The set of surgery duration groups.
$H_n = \{H_{1,n}, H_{2,n}, ..., H_{h,n}\}$	The allowed surgery duration groups combinations, with each $H_{h,n}$ being a list of n numbers of g.
u	Undertime
o	Overtime
b_C	Utilization of a surgical department C $$b_C = \frac{R_t}{A} \cdot 100\% \qquad (1)$$
b	Utilization used in this research $$b = \frac{\left(\sum_{n=1}^{i} d_n\right) + (i-1)t}{S} \cdot 100\% \qquad (2)$$ *A complete explanation of the definition of utilization has been given in Section 4.3*
The expected overtime	$\mathbb{E}[o] = \sum_n \mathbb{P}(n)\mathbb{E}_n[o]$
The expected undertime	$\mathbb{E}[u] = \sum_n \mathbb{P}(n)\mathbb{E}_n[u]$
The expected utilization	$\mathbb{E}[b] = \sum_n \mathbb{P}(n)\mathbb{E}_n[b]$
A	Allocated OR-time
R	Used OR-time : The sum of durations of all surgeries executed by a surgical department throughout a year, excluding the turnover time $$R = \sum_{m=1}^{j} \left(\sum_{n=1}^{i} d_{n,m} \right) \qquad (3)$$
R_t	Used OR-time including overtime and turnover time. $$R_t = \sum_{m=1}^{j} \left(\sum_{n=1}^{i} d_{n,m} + (i-1)t \right) \qquad (4)$$

1.6 Purpose

As mentioned in the introduction, this research focuses on the best session length for each surgical department. So, the purpose of this research is to determine whether different session lengths per surgical department can contribute to a better OR utilization of the departments. In this section, we explain the motivation of this research with an example.

1.7 Problem statement

Many surgical departments of VUmc are asking for (more) longer sessions, because they claim that there is an increase in surgical cases that have a long duration, and a decrease in cases of short duration. With an example we explain the problem statement, and how this research has arose.

Example 1.1. Take surgical department Urology (URO), and suppose they conduct two surgeries per session. They say that they need longer sessions in order to schedule an additional surgeries on that session, because they experience undertime otherwise. In other words, with the current session length, department URO can not execute three surgeries without overtime due to planning.
This is why we want to know what the best session length per surgical department is.

Now, suppose URO has three sessions of eight hours, and on each of those sessions two hours are left after planning two surgeries. Also, suppose they do not have a surgery with an average duration of two hours, so they have undertime. When we would have given URO four sessions of six hours, they could have planned two extra surgeries without any over- or undertime due to planning.
Another possibility, for example, could be to assign one longer session of 10 hours, and two of 7 hours. Then, it might be possible to schedule 3 surgeries on the session of 10 hours.

Concluding, the target is to determine the best combination of session length per surgical department. Our analysis have been based on historical data of surgeries (such as the duration).

2 Research design

This chapter gives a more precise description of the research. We present the research objective and questions, as well as other information needed for conducting this research.

2.1 Research objective

Given the problem statement in Sections 1.6 and 1.7, we have composed the following research objective:

> *Do variable session lengths per surgical department contribute to a better OR utilization?*

2.2 Research questions

To be able to answer the research objective we have to know what (combination of) session length(s) maximizes the OR-utilization for each surgical department. When researching this we want to take into account the regulations about the working hours of the staff, and the number of available ORs. In the end, we want to obtain a session distribution which provides each surgical department with the best OR utilization. Therefore we have the following research questions:

1. *What is the optimal session length per surgical department, when they can have only one length?*
2. *What is the optimal blueprint design concerning the answer of research question 1?*
3. *What is the optimal combination of session lengths per surgical department, when we allow multiple lengths?*

In Section 1.4 we have given that VUmc currently has one regular session length and that some surgical departments sometimes get a longer session. Our research focuses on the option of all departments having their own (optimal) session length. First, we will only allow one length per department, and we want to know which length is optimal per department. Once we have answered that, we want to combine the optimal session of each department into a new blueprint. Lastly, we will take a look at the possibilities when allowing a surgical department to have multiple different lengths.

2.3 Research strategy

Here we have describe how the research questions have been approached, in other words the strategy we have pursued.

In order to properly answer the first research question, we have to determine what we mean by an optimal session. Therefore, we will formulate several criteria. First, we need to know the current situation, in other words, the way the session distribution is composed now, some of the planning rules, the session time allocation, and information on over- and undertime. We also need information of the mix of surgeries of each surgical department. We have approached this in two different ways. First, with a medical point of view, i.e. what kind of surgeries a department performs. Secondly, only by duration: we have analyzed the surgery time frequencies. In both cases, we have studied the expected under- and overtime. Mapping these expectations, we are able to analyze what session length suits a department best. In our research we take into account the options of longer and sorter sessions than the current typical 465 minutes, or even a combination of short and long sessions, while we look at the currently different amount of surgeries executed by a surgical department per session.

We have started with a small scale pilot on two surgical departments. We have chosen Neurosurgery and Urology. The first one has already been assigned longer sessions, as opposed to the second one. So, it could be interesting to compare these two. It will also help to understand the data. Both ways of analyzing the type of operations per department have been based on data of surgeries realized in 2015. This data is discussed in the next chapter.

An extensive data analysis (Section 3), has lead to a model that should determine the best session length for all departments. After we have explored the (combination of) session lengths of all the departments, we aim to design a new allocation of OR blocks. In other words, a new blueprint based on different session lengths. Here, different constraints will have to be met, with at least the regulations of the working hours of the staff, and the amount of ORs available to get a realistic solution.

Last, while answering the research questions we have looked at the utilization when using those answers in practice.

2.4 Research material

The data we have used is data about all elective surgeries realized in 2015. This data has been preworked by VUmc to make the session distribution of 2017, so it has saved us some time in cleaning and manipulating the data. For each of the surgical departments we have information such as the duration of a surgery (the start and finishing times are given), as well as the length of a programmed session (so we can calculate over- and undertime). More details on the data are given in Chapter 3.

Next to the quantitative data, we want to complement it with qualitative data using interviews with a variety of people, such as planners from different hospitals. We will also need the blueprint of 2015 and 2017, these are the time allocations per department for the years 2015 and 2017 respectively (see Section 2.5.1). It only provides information on the number of hours allocated on an OR assigned to each department per day of the year. The *planning bureaus* of the surgical departments have the job to schedule the surgeries in departments assigned OR-time. The realized surgery durations of 2015 have been our base when determining new session lengths.

2.5 Current situation

This section explains the session distribution of 2015 and 2017 (which are not the same), and gives the corresponding OR-time allocation of each surgical department. We also have given some regulations and agreements that the best session lengths and a new blueprint have to meet.

2.5.1 Blueprint and session distribution

The so-called *blueprint* is the distribution of OR-time per surgical department. This is a schedule containing the amount and length of sessions assigned to each surgical department per day. The session allocation is not a surgery schedule yet. Surgeries are scheduled by the planning bureau of a surgical department.

Until 2017, there has been a distinction between the *blueprint* and *session distribution*. The blueprint used to consist of five periods, with a period being the OR-time distribution of one week. These periods are put in a specific sequence to get the schedule for the whole year, the *session distribution*. Most of the time this sequence is 1-2-3-4-1-2-... etc. In the months July, August, and December, the sequence will be 5-5-5-..., because period 5 is composed for holiday periods in which less staff is available. This is how the session distribution arises. After the session distribution in known, the surgical departments can start to schedule surgeries on their assigned session time (we will not concern that in this research).

Figure 3 gives period 1 of the blueprint of 2015, containing only the surgical departments we studied at in this research.

Blueprint 2015
Period 1

begintime: **08.00**
endtime: **15.45** | **17:45** | **19:45**

OK 6A	Monday	Tuesday	Wednesda	Thursday	Friday	total sessions this week	times this period occurs in 2015	total sessions o period 1
CCH	2,3	2,3	1,3	2,3	2,0	10,3	10	103
LON			1,0	1,0		2,0	10	20
ORT	1,0	1,0	1,0	1,0	1,0	5,0	10	50
HLK	3,0	2,0	3,0	2,0	3,0	13,0	10	130
NCH	1,0	2,0	1,0	2,0	1,0	7,0	10	70
GYN	1,0	1,0	1,0	1,0	1,0	5,0	10	50
URO	1,0	1,0			1,0	3,0	10	30
GENDER						0,0	10	0
Totaal	**10,3**	**10,3**	**9,3**	**10,3**	**10,3**	**45,3**	**10**	**453**
OK 2D	OK	OK	OK	OK	OK			
MKA		1,0	1,0			2,0	10	20
PCH	1,0		1,0	1,0	1,0	4,0	10	40
KNO	2,0	1,5	2,0	2,0	1,0	8,5	10	85
OOG		1,0		1,0	1,0	3,0	10	30
KCH		1,0				1,0	10	10
URK			1,0		1,0	2,0	10	20
Totaal	**5,0**	**6,5**	**7,0**	**6,0**	**6,0**	**20,5**	**10**	**205**
Totaal	**15,3**	**16,8**	**16,3**	**16,3**	**16,3**	**65,8**		

Figure 3: Period 1 of blueprint 2015

From 2017, the distinction between session distribution and blueprint have been abandoned. Now, the session distribution is composed immediately without using five periods. This way, it is easier to regard the holidays and training days of the staff.

Figure 4 gives the first two weeks of the blueprint (or session distribution) of 2017. The numbers and letters in figure 4 have the following meaning:

Holiday period
Session subject to change
L1 = late start 1 (09.00 - 15.45)
L2= late start 2 (09.30 - 15.45)
N = normal (08.00 - 15.45)
KL = long session 1 (08.00 - 17.30)
L = long session 2 (08.00 - 17.45)
EL = extra long session (08.00 - 19.45)
KA = Child anesthesia

When an '1' is given in Figure 4, the department gets on that day one OR with a session of regular length. An '1L' means one long session is assigned, and a '2' means, two ORs of regular length have been assigned. When a departments has a '2L' assigned, one regular session is assigned and one long session, etc. A '2L' never means that two long sessions are assigned.

Session distribution OR 2017

		Holiday period									
Weeknummer	Floor	Week 1					Week 2				
Date		2-jan	3-jan	4-jan	5-jan	6-jan	9-jan	10-jan	11-jan	12-jan	13-jan
Late start			L1								
CCH	6	1L	1L	1L	1L	1L	2L	2L	2L	2L	2L
LON	6			1	1				1	1	
ORT	6	1KA		1			1KA	1	1		
HLK	6	2	2	2	2	2	3	3	3	3KL	3
NCH	6	1KL	1	1	1	2	1	1	1	1	2
GYN	6	1	1	1	1	1	1	1	1	1	1
URO	6	1	1			1	1	1			1
GENDER (op 6)	6				1KL					1	
GENDER (op 2)	2				1		1			1	
MKA	2		1KA					1	1	1	
PCH	2	1				1KA	1	1KA			1
KNO	2	1	1EL	1	1	2KA	2	1EL	2	1	2KA
OOG	2	1LO		1	1KA		1	1	1KA	2C	1LO
KCH	2		1KA					1KA		1KA	
URK	2			1KA					1KA		1KA
Flexibele OK tijd	W		1				1				
Spoed OK	6	1	1	1	1	1	1	1	1	1	1

Figure 4: First two weeks of session distribution 2017

The amount of session time each department has been allocated in the upcoming year, is determined using data of the realized surgeries in the previous year. Hence, the OR-time allocation of 2017 is presented in October of 2016 using the realized surgery duration in 2015. This is due to the fact that we want to base the time allocation per whole year, and the complete set of historical data of 2016 will not be available until the start of 2017.
The model used to determine the OR-time allocation for 2017 works as follows. First, the utilization is calculated for each department. The definition of utilization will be given in Section 4.3. Each department retains 90% of the OR-time allocated in 2015. The remaining time will be redistributed over the surgical departments, and one part will be saved for a flex OR. The flex OR will not be assigned to any department in the blueprint, but they can appeal to it. When redistributing OR-time, departments with a high utilization will get likely get more time then the departments with a lower utilization. Also, the goal of a department for 2017 have been taken into account. A department that has a higher probability in growing, or conducting more or new kind of surgeries will get more OR-time.

In mathematical analysis, we have used the OR-time allocation of 2015. The allocation of 2017 is used when determining the new blueprint based on the best combination of session lengths which will provide a better utilization.

Appendix A.2 states an explanation of the software used by VUmc to control the OR-schedule.

As explained earlier, most surgical department get assigned the regular session length of 465 minutes assigned, and some departments have been assigned some longer sessions. Department CCH has a long session of 585 minutes every day, HLK a session of 570 minutes on Thursday every week, and NCH one on Monday every two weeks. The extra long session of 705 minutes has only been assigned to KNO and PCH. They get the extra long session every week respectively every four weeks.

In the next section we have given the OR-time allocations of 2015 and 2017.

2.5.1.1 OR-time allocation

As discussed in the previous section, we will use the OR-time allocation of 2017. For our data analysis we need the time allocation of 2015. Below we have given both:

Surgical Department	Time allocated 2015	Time allocated 2017
CCH	210,421	220,260
LON	43,245	50,400
ORT	105,090	67,740
HLK	270,180	312,180
NCH	129,165	146,646
GYN	88,590	117,240
URO	59,760	72,360
GENDER	9,765	47,340
MKA	39,525	52,560
PCH	80,910	66,480
KNO	169,493	202,620
OOG	79,515	106,020
KCH	30,690	41,220
URK	35,340	42,060
Total	1,351,688	1,550,400

Table 2: OR-time allocation of 2015 and 2017 (in minutes)

Some remarks:

In September of 2015, VUmc suffered flooding due to a broken water-pipe in the neighborhood of the hospital[1]. It is not certain that we have the correct data of this period, because of the possibility of insufficient documentation or incomplete data due to the circumstances. The OR-time allocated (and used) is adjusted in the period of 7-9-2015 until 26-9-2015, we have excluded this period.
The OR-time allocation of 2017 has not been adjusted for that period. Therefore, we can not compare the assigned OR-time of 2015 and 2017.

We see a large cap between the OR-time of the surgical department Gender in 2015 and 2017. This is because of the fact that Gender has grown a lot during 2015, and has been allocated session time of other departments. In 2017, they have been given their own session time.

[1] http://nos.nl/artikel/2056529-grote-problemen-bij-vumc-amsterdam-na-waterlek.html

2.5.2 Rules and agreements

The main goal of this research is to determine the best session length for each surgical department. When we want to design a new session distribution based on these session lengths, we need to meet certain rules. We have used the planning rules 'Planningsregels Dienst Operatiekamers VUmc' [21] as a guideline. These rules have been updated last in 2014. Therefore, we have only listed the ones still relevant regarding the blueprint.

- The aim is to announce the blueprint for the coming year in October.
- In the blueprint, an OR is only assigned to one surgical department.
- Everyday, at least one OR is assigned for emergency surgeries. When this one is occupied, elective cases might have to be cancelled or moved.
- The current session lengths are:
  - Late start 1: 09.00-15.45 (405 minutes)
  - Late start 2: 09.30-15.45 (375 minutes)
  - Regular: 08.00-15.45 (465 minutes)
  - Long session 1: 08.00-17.30 (570 minutes)
  - Long session 2: 08.00-17.45 (585 minutes)
  - Extra long session: 08.00-19.45 (705 minutes)

  Some days have educational meetings in the morning. On those days the elective program starts not at 08h00 but on 09h00 or 09h30. This is what we call a 'late start'.
  Also, some of the surgical departments have surgeries that take more time then the regular session length of 465 minutes. These departments, such as cardio surgery (CCH) and surgery (HLK), have been given longer sessions.
- The standard turnover time is 20 minutes.
- Permission for an additional long session is given by the session manager responsible for ensuring that a team is available and it is logistically possible.
- The session distribution has a session with a proviso, every day. This session is canceled first if there are circumstances that make it impossible for the planned program to continue otherwise.
- When a surgical department fails to use 90% of the allocated session length or exceeds 100% usage, we speak of failure to comply with the set norm.
- The utilization of a surgical department should be at least 85% at the end of the year.
  See Section 4.3.

Next to these rules, we also have to take into account some other requirements. We will discuss these in the next few paragraphs.

2.5.2.1 Working hours

To get a realistic solution we should take into account the requirements regarding the working hours of the staff. We have used the regulations of the CAO of VUmc [23].

The most important regulations are:

- The complete workload sums up to 1,872 hours per year, which corresponds to 36 hours per week (5 days).
- The acknowledged Dutch holidays are: New Year's Day, the second day of Easter, Ascension Day, the second day of Pentecost, and (two days of) Christmas.
  For each holiday that takes place on a Saturday or Sunday, the working hours will be lowered with 7.2 hours as compensation.
- In consultation with the employer, an additional amount of hours up to 208 can be added,
- or be lowered with maximally 184 hours.
- A regular working day (Monday up to including Friday) starts at 07h00 earliest and ends on 18h00 latest.

2.5.2.2 Amount of ORs available

In Section 1.3.1 we have described the ORs available at VUmc. When making the session distribution we have to take into account that only fifteen ORs are available for elective programs each day. We can not distribute more ORs per day than available.

2.5.2.3 Bed capacity

There are not infinitely many beds available for patients who just had surgery (whether or not they have to stay a night in the hospital). Therefore, in the OR schedule the outflow of patients should not exceed the amount of available beds. The way the session distribution is made could contribute to a good outflow of patients. However, in this research we are not looking at the actual planning of surgeries but just at the overall session distribution, so we have decided to neglect this condition for now. Moreover, multiple projects of VUmc are focused on this matter.

2.6 Situation in other UMCs

It is interesting to know something about OR-allocation strategies of other hospitals. We decided that we want to know more about some of the other UMCs in the Netherlands, because it is most likely they come across the same difficulties as VUmc. We have contacted LUMC (Leiden), AMC (Amsterdam), and Erasmus MC (Rotterdam). These three hospitals make the OR blueprint similar to VUmc. They run into some of the same issues, but they pursue different ways of handling and solving these issues. In this section, we will discuss what we have learned from LUMC, AMC, and Erasmus MC.

2.6.1 LUMC

With LUMC we had mainly contact on whether they do something with variable or flexible session lengths. Variable meaning that not every surgical department has the same regular session length. Flexible session lengths, means not having a fixed session length, it could change during the day when less or more OR time is needed.
LUMC is doing a lot of new research and projects concerning their OR facility. A few years ago, they have conducted a research about using half sessions instead of only the regular 7.5 hour sessions, which did not induce a better OR utilization. Some other conclusions of this research were mostly practical objections regarding the working schedule and working hours of the staff.

2.6.2 AMC

VUmc and AMC work together on many projects, but still do their own (unique) research as well. At AMC, they are aware that OR utilization, cancellation of surgeries, and overtime have a close connection with each other. For example, when you want to get a higher utilization you need to lower the amount of cancellations. However, a higher utilization comes hand-in-hand with more overtime.
Next to the session distribution, AMC has one room that is reserved for elective surgeries in the morning, and in the afternoon for emergencies. Doing this, they do not want overtime on the morning schedule, so there has to be good knowledge about the predictability of surgery times.
Recently, AMC introduced flexible working hours. Each day, a few staff members will be assigned to work overtime when needed. This way, less cancellations will occur.

In Appendix A.3, a review of the meeting at AMC can be found.

2.6.3 ErasmusMC

Erasmus MC also works with a regular session length with an exception for some surgical departments who get a scheduled 'overtime room'. Erasmus MC is one of the UMCs already using historical surgery data intensively. Using historical data such as the time a surgery took to be completed, and which surgeon did the procedure, a better expectation of the actual surgery duration can be made. Therefore less under- and overtime will probably occur.

In Appendix A.4, a review of the meeting can be found.

3 The surgery data

In this chapter we discuss the data we are using in this research.

3.1 Data description

The data we have received from VUmc to use in this research, is all data about surgeries executed in 2015. Recall, we have decided to use only the elective cases, because we want to focus on the session distribution which does not yet take into account the possible emergency operations. Also, the period 7-9-2015 until 26-9-2915 (Section 2.5.1.1) has been excluded.
We have two sets of data. One contains information on the surgeries conducted per surgical department, with information such as the surgery durations, in which OR the surgeries were executed, on the overtime, and it contains patient and surgery numbers. The second set gives all operations that were needed to complete each surgery (each surgery exists of one or multiple operations). Unfortunately, this data set is not yet in the same format as the first (already pre-worked) data-set. However, because we know all surgery and patient numbers, we can filter the correct lines of data in the second set that match the ones in the first set.

The rest of this chapter gives more detail on the information the data contains. We need this information to get a good understanding of the current situation, and what should change to get a higher OR utilization. First, we will explain how we aim to achieve this.

3.2 Course of action

We have started the analysis on the data by obtaining some general information. This was information such as how many surgeries each surgical department executed, how much surgeries per session were executed, and the average surgery duration. We have also calculated the overtime and undertime in 2015 was calculated.

After obtaining this information, we decided that we want to get more knowledge on the type of surgeries a surgical department conducts, which we need to know before we can determine the best session lengths. We have to know what kind of surgeries could possibly be scheduled per session. We have started this research with the intention to cluster the surgeries into types in two different ways. First in a medical point of view, secondly by surgery duration. While the research progressed, we decided that the first way was not opportune due to an other project at VUmc that is exploring this issues. Also, pursuing this way we would probably have to develop some planning rules (for example, advise which surgery types should be scheduled together), and management did not want us to do that. We merely want to look at the best length of a session for a department. However, we spent many hours on clustering surgeries on a medical point of view. Therefore, we have shortly elaborate on this part of the research in Section 5.1.

In the previous section, we have explained that the second data-set is not yet completely correct. In other words, the set might contain surgeries that are not assigned to the correct surgical department, or surgeries are denoted as elective but were actually emergency surgeries, and vice versa. So we needed to filter the second data-set (containing all operations) on the correct (elective) patient and surgery numbers. A patient can have multiple surgeries in a year, but a surgery conducted always gets an unique number. Therefore, we combined the patient and surgery numbers belonging to the same surgery. The first data set contains all correct elective cases. Taking the patient-surgery combinations from that data-set we can get the same combinations of the second data set. After doing so, we can combine the operations belonging to the same surgery. Then we have determined the unique combinations, and how often they have occurred. The operation codes exist of six letters, and we have analyzed the unique combinations with and without these letters. We have executed this for Neurosurgery (NCH) and Urology (URO) using Microsoft Excel (see Section 5.1).

The main part of the research is based on the second method of clustering surgeries, clustering merely by duration. We have used the frequency of surgery times within each surgical department, and grouped these by intervals of 20 minutes. We have chosen to use intervals of 20 minutes, because it was a logical choice in view of the 20 minutes turnover time.
Using statistical software R, we have made histograms of the surgery times, and we have tried to fit probability distributions on the surgery durations of each department. This will be further explained in Section 5.2.

3.3 General information about the current situation

Here we give an overview of the OR-time allocation in 2015 versus the actual time used. We give the amount of sessions distributed in the blueprint, and the actual used sessions. Also, the amount of surgeries conducted in 2015 is given. Note, we have not specified the session length, but have only looked at the amount of sessions. Further, Gender has gotten more sessions throughout the year 2015 than were assigned in advance. Most of the sessions given to Gender, were originally assigned to PCH.

	A	R	R_t	S_A	S_R	$\frac{S_R}{S_A}$	i
CCH	210,421	159,803	164,326	414	363	0.88	520
Gender	9,765	31,935	33,087	23	96	n/a	182
GYN	88,590	73,552	78,481	197	176	0.89	510
HLK	270,180	255,241	268,755	610	595	0.98	1,318
KCH	30,690	22,694	27,255	69	66	0.96	7307
KNO	169,493	138,755	148,879	358	340	0.95	1,092
LON	43,245	33,681	35,574	98	84	0.86	170
MKA	39,525	45,795	48,277	88	84	0.95	296
NCH	129,165	118,592	122,185	290	275	0.95	449
OOG	79,515	58,218	67,310	140	171	1.22	645
ORT	105,090	97,640	103,762	238	223	0.94	585
PCH	80,910	56,036	59,423	178	145	0.81	353
URK	35,340	27,204	29,836	81	75	0.93	259
URO	59,760	53,906	56,719	133	127	0.95	282
TOTAL				2,917	2,820		

Table 3: Allocated time and sessions for 2015, versus realized in 2015.

Where:

A = the allocated OR-time (see Section 2.5.1.1).
R = sum of realized surgery time
R_t = realized time including turnover times (see Section 3.3.1)
S_A = amount of sessions allocated
S_R = amount of sessions used
i = amount of surgeries

S_A, S_R are the amount of sessions, the length is not specified.

We have compared the used OR-time versus the allocated OR-time to see whether surgical departments accurately use their OR-time. When there is a large deviation between the assigned time, and the realized time we can conclude that the program of the departments is insufficient. Note, that almost all surgical departments use more than 85% of their allocated sessions. However, this does not yet say enough about the performance of a surgical department. We do have to look at the utilization, because a surgical department could have used, for example, only 60% of the assigned session length.

The new amount of sessions a department gets assigned depends on the new session length, and is determined using the OR-time allocation of 2017. Table 4 gives the OR-time of each departments in 2017, and the amount of surgeries allocated in the blueprint of 2017. Note, this is based on the blueprint released at the start of 2017, it is possible that throughout the 2017 some changes are made in the actual session distribution given to the planning bureaus.

	A	S_A
CCH	220,290	469
Gender	47,355	100
GYN	117,210	248
HLK	312,195	664
KCH	41,190	88
KNO	202,620	431
LON	50,370	106
MKA	52,545	112
NCH	146,625	311
OOG	106,020	225
ORT	67,710	144
PCH	66,465	142
URK	42,060	89
URO	72,330	154
TOTAL		3,283

Table 4: Allocated time and sessions for 2017.

Recall, the allocated OR-time of 2015 was adjusted for a particular period, therefore we can not compare the amount of sessions in 2015 and 2017.

3.3.1 Allocated versus realized

Appendix C.1 contains graphs for each surgical department to show the OR-time allocated versus the time actually used. We have done this per week. The realized time is the sum of all surgeries performed, including the time between these surgeries on one OR. The time between the surgeries is not the actual turnover time, because it is possible that in our data a large turnover time occurs due to the fact we have only looked at elective cases and excluded the emergency ones. However, we did include the times between surgeries on the same OR, so that we get a better image of how well the surgical departments have performed because the allocated time is included turnover times.

Some observations based on the graphs:

- Most of the departments used their allocated time very well.
- We can not make any conclusions based on the graph of Gender, because throughout the year more OR-time was assigned than first allocated.
- PCH used far less time than allocated. However, in Section 4.3 we see that PCH has a good utilization. This is due to the fact, OR-time of PCH is given to Gender.
- LON, and MKA have both some large outliers.
- In the graph of OOG we do not see anything ordinary. However, in Table 3 we see that OOG used more session than they had been assigned. We have to conclude that we probably miscalculated something, or the data is incorrect.

3.4 Number of surgeries per OR-session

It is possible to schedule surgeries per session as long as there is time left and overtime does not occur due to planning. Therefore, one or more surgeries are scheduled per session, depending on the surgery times. We have calculated how often a surgical department conducts a specific number of surgeries in a session. Table 5 gives the frequency that a number of surgeries is performed per session, and Table 6 gives the probability that the amount occurs. Here, we have only taken into account the surgeries which were executed in the departments own sessions. This will give a slight distortion, because we have excluded some elective cases.

Department	# surgeries	# sessions	1	2	3	4	5	6	7	8	9	10
CCH	519	363	209	152	2	0	0	0	0	0	0	0
Gender	177	96	43	32	15	5	1	0	0	0	0	0
GYN	500	176	15	56	63	30	9	2	1	0	0	0
HLK	1,293	595	143	280	116	39	16	1	0	0	0	0
KCH	303	66	2	1	14	17	13	10	8	1	0	0
KNO	1,087	340	56	89	57	62	43	18	11	2	2	0
LON	147	84	30	45	9	0	0	0	0	0	0	0
MKA	250	84	7	12	42	22	1	0	0	0	0	0
NCH	446	275	139	110	18	7	1	0	0	0	0	0
OOG	645	171	3	20	41	65	33	9	0	0	0	0
ORT	583	223	46	46	81	48	2	0	0	0	0	0
PCH	349	145	43	26	51	24	1	0	0	0	0	0
URK	253	75	3	10	31	18	13	0	0	0	0	0
URO	268	127	27	72	20	5	2	0	1	0	0	0

Table 5: Amount of days with i surgeries in a session

Department	1	2	3	4	5	6	7	8	9	10
CCH	0.58	0.42	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gender	0.45	0.33	0.16	0.05	0.01	0.00	0.00	0.00	0.00	0.00
GYN	0.09	0.32	0.36	0.17	0.05	0.01	0.01	0.00	0.00	0.00
HLK	0.24	0.47	0.19	0.07	0.03	0.00	0.00	0.00	0.00	0.00
KCH	0.03	0.02	0.21	0.26	0.20	0.15	0.12	0.02	0.00	0.00
KNO	0.16	0.26	0.17	0.18	0.13	0.05	0.03	0.01	0.01	0.00
LON	0.36	0.54	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MKA	0.08	0.14	0.50	0.26	0.01	0.00	0.00	0.00	0.00	0.00
NCH	0.51	0.40	0.07	0.03	0.00	0.00	0.00	0.00	0.00	0.00
OOG	0.02	0.12	0.24	0.38	0.19	0.05	0.00	0.00	0.00	0.00
ORT	0.21	0.21	0.36	0.22	0.01	0.00	0.00	0.00	0.00	0.00
PCH	0.30	0.18	0.35	0.17	0.01	0.00	0.00	0.00	0.00	0.00
URK	0.04	0.13	0.41	0.24	0.17	0.00	0.00	0.00	0.00	0.00
URO	0.21	0.57	0.16	0.04	0.02	0.00	0.01	0.00	0.00	0.00

Table 6: Chance of i surgeries in a session

Remark: We have given the probabilities with two decimal places. Therefore, the sum of the probabilities of a surgical department might not seem to add up to be exactly equal to 1.

In our simulation, we have decided to only take into account the amount of surgeries per session with a probability of more than 0.10. A probability lower than 0.10 implies it is likely that the amount in question will not occur in the next year. Excluding the amount of surgeries with a probability lower than 0.10 will also simplify our model.
For surgical departments GYN and MKA we have made an exception. These two departments have an probability of less than 0.10 of a session with one surgery, however, if we do not incorporate the option of one surgery per session we can not execute (very) long surgeries without a large amount of overtime. This will be explained further in Section 6.3.

3.4.1 Number of surgeries a surgical department is allowed to schedule

Table 6 states in green the number of surgeries per session that will be considered for each surgical department. We have considered these as the only possible amounts, thus the probabilities in Table 6 have to be re-normalized. The re-normalized probabilities have been given in the table below.

Department	1	2	3	4	5	6	7
CCH	0.58	0.42	0.00	0.00	0.00	0.00	0.00
Gender	0.48	0.36	0.17	0.00	0.00	0.00	0.00
GYN	0.09	0.34	0.38	0.18	0.00	0.00	0.00
HLK	0.27	0.52	0.22	0.00	0.00	0.00	0.00
KCH	0.00	0.00	0.23	0.27	0.21	0.16	0.13
KNO	0.18	0.29	0.19	0.20	0.14	0.00	0.00
LON	0.36	0.54	0.11	0.00	0.00	0.00	0.00
MKA	0.08	0.14	0.51	0.27	0.00	0.00	0.00
NCH	0.56	0.44	0.00	0.00	0.00	0.00	0.00
OOG	0.00	0.13	0.26	0.41	0.21	0.00	0.00
ORT	0.21	0.21	0.37	0.22	0.00	0.00	0.00
PCH	0.30	0.18	0.35	0.17	0.00	0.00	0.00
URK	0.00	0.14	0.43	0.25	0.18	0.00	0.00
URO	0.23	0.61	0.17	0.00	0.00	0.00	0.00

Table 7: Re-normalized chance of i surgeries in a session

Remark: We have given the probabilities with two decimal places. Therefore, the sum of the probabilities of a surgical department might not seem to add up to be exactly equal to 1.

3.4.2 Average number of surgeries per day

Next to the amount of surgeries per session, we have also calculated the average amount of surgeries executed on each day of the week. The reason is that some departments prefer to perform multiple small surgeries on a specific day, or have one or two large surgeries on a same day each week. This insight will help us when making a new blueprint, to make sure it is not a complete change with a current one. Again, we have only looked at the surgeries executed on the departments own sessions.

Where:
i is the amount of surgeries,
d is the surgery duration.

		Monday	Tuesday	Wednesday	Thursday	Friday
CCH	Average i	1.40	1.50	1.46	1.37	1.44
	Average d	325.11	313.75	300.27	299.24	296.44
Gender	Average i	1.86	1.44	2.00	1.89	1.76
	Average d	201.46	153.15	156.61	178.36	194.83
GYN	Average i	3.16	3.29	2.59	2.09	3.22
	Average d	135.49	122.79	160.07	184.07	125.34
HLK	Average i	2.56	2.00	2.47	1.86	2.05
	Average d	165.35	218.16	177.76	229.06	196.36
KCH	Average i	1.00	4.08	0.00	6.24	0.00
	Average d	27.00	83.19	0.00	55.64	0.00
KNO	Average i	3.84	2.39	3.66	2.81	3.03
	Average d	98.50	265.20	93.72	141.42	119.76
LON	Average i	0.00	0.00	1.77	1.64	0.00
	Average d	0.00	0.00	210.59	214.62	0.00
MKA	Average i	0.00	2.83	3.17	0.00	0.00
	Average d	0.00	159.37	143.22	0.00	0.00
NCH	Average i	1.38	1.59	1.42	2.18	1.42
	Average d	327.90	262.99	301.07	191.47	312.88
OOG	Average i	3.48	3.63	3.24	3.94	4.22
	Average d	83.08	111.58	103.89	98.75	75.03
ORT	Average i	3.32	3.07	1.47	2.62	2.34
	Average d	126.41	139.94	319.11	163.82	187.29
PCH	Average i	2.43	3.05	1.69	1.65	2.92
	Average d	157.49	130.23	263.04	190.11	120.78
URK	Average i	0.00	0.00	3.16	4.00	3.52
	Average d	0.00	0.00	112.66	96.94	92.89
URO	Average i	2.02	2.39	2.75	1.00	1.89
	Average d	217.04	171.40	117.97	240.00	215.58

Table 8: Average amount of surgeries per day and average duration of such a surgery.

We have visualized this data for each surgical department, cf. Appendix C.2. Below, we have given these graphs for departments NCH and URO, as an example. Of course, on days with a high average surgery duration, less surgeries are executed than on a day with a low average surgery duration.

The next table gives the average amount of surgeries on a day, and the average surgery duration of each departments in 2015:

	Average i	Average d
CCH	1.43	307.45
Gender	1.84	176.33
GYN	2.84	144.22
HLK	2.18	194.63
KCH	4.59	73.37
KNO	3.20	127.26
LON	1.71	212.48
MKA	2.98	152.00
NCH	1.62	265.20
OOG	3.77	90.26
ORT	2.61	167.02
PCH	2.41	159.12
URK	3.37	104.09
URO	2.09	195.19

Table 9: Average number of surgeries and average duration of such a surgery.

Note, we have only taken into account the surgeries executed on the surgical departments own sessions, therefore there is a small deviation in the average duration of a surgery compared to the average duration we get from our model, see Section 6. This is because in the model, we have made empirical probability distributions based on all elective surgery durations that have occured.

3.5 Overtime and undertime

One of the goals of this research is to minimize the amount of over- and undertime by allocating new session lengths. Therefore, we first need to know how much over- and undertime each surgical department had in 2015.

Recall from Section 1.4, overtime is defined as the time used to complete a surgery after the end of the planned session length. Undertime has been defines as the unused time between the end of the last surgery and the end of the planned session length.
In the data-sets we have been granted, the overtime has been given in a separated column. Figure 10 gives the total amount of overtime, and the average overtime for each surgical department.

	Total overtime (in minutes)	Occurrence	Average overtime over occurrence	Average overtime over all surgeries	Percentage of surgeries with overtime
CCH	6,557	95	69.02	12.63	18 %
Gender	1,440	31	46.45	8.14	18 %
GYN	3,489	80	43.61	6.98	16 %
HLK	19,049	305	62.46	14.73	24 %
KCH	797	29	27.48	2.63	10%
KNO	4,298	100	42.98	3.95	9 %
LON	2,059	42	49.02	14.01	29 %
MKA	4,007	57	70.30	16.03	23 %
NCH	7,967	126	63.23	17.86	28 %
OOG	1,251	38	32.92	1.94	6 %
ORT	7,477	121	61.79	12.83	21 %
PCH	3,529	56	63.02	10.11	16 %
URK	398	13	30.62	1.57	5 %
URO	1,865	48	38.85	9.96	18 %

Table 10: Analysis of the overtime per surgical department.

Figure 10 shows that HLK has many minutes of overtime. This suggests that the current session length of HLK is insufficient. However, the overage overtime over all surgeries of HLK is not the highest one. That is for the surgical department NCH. We can also see that when overtime occurs for department MKA, it has the highest average overtime. Therefore, for the same reasons mentioned, for NCH, MKA, CCH, and ORT the current session length(s) assigned is probably also insufficient.

The amount of undertime is harder to calculated. Since we are only considering the conducted elective cases and not the emergency surgeries, we could have a large undertime due to the fact that an emergency case had to be performed on the elective OR-time of a surgical department. Another problem occurs when an department conducts a surgery on the OR-time of another department. To whom should we assign the undertime in this case?
However, we do want to know more about the undertime per department. Under certain assumptions, we have come up with a way to get some insight in the amount of undertime for a department. Our method for calculating the undertime is as follows:

Algorithm 3.1.

1. Consider the last surgery of an OR.
2. Did it end before the end of the assigned session length?
   - 2a. No: no undertime
   - 2b. Yes: Was the surgery conducted on the departments own allocated session?
     - 2b1. No: We do not have enough information, therefore we do not include this undertime for any surgical department.
     - 2b2. Yes: The time between the end of the surgery and the end of the session, is considered as undertime for the department to whom the session was allocated to.

Figure 11 gives the total amount of undertime, and the average undertime for each surgical department. Keep in mind this is most likely not the real undertime, but it gives an indication of it.

	Total undertime (in minutes)	Occurrence	Average undertime over occurrence	Average undertime over all surgeries	Percentage of surgeries with undertime
CCH	31,971	259	123.44	61.60	50 %
Gender	1,949	43	45.33	11.01	24 %
GYN	5,135	86	59.71	10.27	17 %
HLK	20,638	273	75.60	15.96	21 %
KCH	2,780	33	84.24	9.17	11 %
KNO	15,146	209	72.47	13.93	19 %
LON	3,929	46	85.41	26.73	31 %
MKA	1,617	28	57.75	6.47	11 %
NCH	12,543	138	90.89	28.12	31 %
OOG	11,430	125	91.44	17.72	19 %
ORT	4,924	92	53.52	8.45	16 %
PCH	4,174	68	61.38	11.96	19 %
URK	3,649	49	74.47	14.42	19 %
URO	4238	70	60.54	15.81	26 %

Table 11: Analysis of the undertime per surgical department.

If the undertime given in Figure 11 was the correct undertime, CCH is either really bad at planning, or the assigned session lengths are not compatible with the surgery durations of CCH. Again, we see for HLK that the current session length is probably not good due to the amount of times undertime occurs. We also see that departments LON and NCH have the most average undertime over all surgeries. We can say with great probability that some adjustments in the session length of LON en NCH is needed.

4 Optimization criteria

We want to find the best or even optimal session length per surgical department. However, when can we state that one session length is preferred over the other? In this chapter, we give some criteria for the optimal session length to meet.

4.1 Literature

In Appendix B. one can find an extensive literature review. Here, we have listed the most important things:

- Surgeries should be scheduled per session in increasing order of duration. In other words, a surgery with a shorter average duration should be scheduled before a surgery with a longer average duration, cf. [7].
- A good understanding of the variation in surgery duration is needed to improve patient care in its broadest sense, cf. [10].
- Longer sessions are better than many short ones [11]. It is also preferable to allow sessions to be no longer than ten hours.
- A higher variability in surgery duration will cause a lower OR utilization, and vice versa. See [15].
- A constant turnover time of 20 minutes is used in many of the articles we have read, e.g. [15], [21].
- The definition of utilization used in articles we have read implies that a larger utilization means more overtime. We have decided to use this definition as well. See Section 4.3.

4.2 Necessary criteria

To be able to evaluate a session length, we have to take into account the undertime, overtime, and the utilization. In this research we determined the expected overtime, expected undertime, and the corresponding expected utilization for different session lengths, see Section 6.

VUmc requires that the OR utilization should be at least 85% for each surgical department. Our aim is to determine the session length for each department with maximum utilization and with minimum overtime. In other words, we want to minimize the expected over- and undertime.
Also, we have to take into account the regulations given in Section 2.5.2. Therefore, we have decided that the session lengths should be between 5 (300 minutes) and 10 hours (600 minutes) (as given in [11]). However, the longest session length assigned to a department at VUmc is almost 12 hours (720 minutes), see Section 1.4. In 2017, it was assigned to plastic surgery (PCH) once every four weeks, and to ear, nose, and throat surgery (KNO) every week. Therefore, for these two surgical departments we have chosen to consider $300 \leq S \leq 720$ minutes.

We have studied three different objectives. First, we have valued overtime and undertime equally. Secondly, we have given overtime a higher weight than undertime. This situation is supposedly more realistic, because overtime comes with more cost (such as extra wages for the staff) than undertime does. As a last objective, we want a utilization of minimal 85% with minimum overtime. Note, minimum overtime or minimum undertime does not necessarily imply a utilization of 85%.

4.3 Utilization

We have come across different ways to define utilization. In this section, we discuss multiple definitions, and motivate which one we have decided to use in this research.

First, we have to make the distinction between OR utilization and utilization of a surgical department. OR utilization shows how well the OR has been used. It is possible that multiple surgical departments did surgery on the OR. In this research we do not take emergency surgeries into account, so most likely only one department did surgery on the OR in question. However, it is possible that a department breaks into the elective program of another department, because a surgery could not be postponed, and their own assigned OR was not available anymore due to some malfunction or an emergency surgery. Therefore, OR utilization does not necessarily have the same meaning as the utilization of the surgical department whom the OR was assigned to.
See Section 1.5 for the meaning of the abbreviations and notations. The definition of elective OR utilization is the following:

$$b_{OR} = \frac{\left(\sum_{n=1}^{i} d_n\right) + (i-1)t_k}{S} \cdot 100\% \tag{5}$$

Looking at Figure 5 below the utilization would be $b_{OR} = \frac{d_1+d_2+d_3+t_1+t_2}{S} \cdot 100\%$,
with $d_1, d_2, d_3 = (d_{3a} + d_{3b})$ elective cases and t_1, t_2 turnover times.

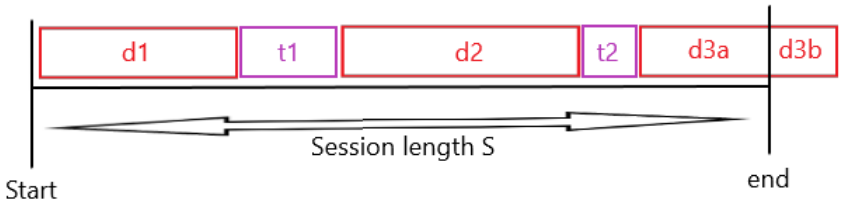


Figure 5: OR-day example

Concerning this definition of OR utilization, we can also decide to include overtime or not. Using Figure 5, the definition is $b_{OR} = \frac{d_1+d_2+(d_{3a}+d_{3b})+t_1+t_2}{S} \cdot 100\%$ when we do include overtime, and otherwise it would be $b_{OR} = \frac{d_1+d_2+d_{3a}+t_1+t_2}{S} \cdot 100\%$, when we do not include overtime. In the first case, utilization could exceed 100%. Therefore, we can conclude that a higher utilization comes with more overtime. A utilization of more than 100% does not seem to be logical, but including overtime when calculating the utilization will implicitly give an understanding of the amount of overtime.

The second way of defining utilization, is the utilization of a surgical department. This measures whether the department is efficiently using their allocated OR time. This can be done throughout the year, to see whether a department needs more time or whether time can be re-assigned to another department. It is especially needed at the end of the year to conclude how well a department has performed. When a surgical department has a high utilization and has the expectation of growth in the next year, it could be decided that they need more OR-time in the next year, or the other way around, they get less OR-time due to having a low utilization.

In the model that was used (see Section 2.5.1) to calculate the OR-time allocation for 2017, the following definition of utilization of a department was used:

$$b_C = \frac{\sum_{m=1}^{j} \sum_{n=1}^{i} (d_{n,m})_{\text{IOT}}}{A \cdot co} \cdot 100\%. \tag{6}$$

with co the correction of the average percentage of surgeries conducted per session time of another department within operating time. This definition of utilization excludes overtime (IOT=*inside operating time*), and turnover time. Using this definition, the utilization of the departments in 2015 were:

Department	Utilization 2015
CCH	72 %
GENDER	79 %
GYN	86 %
HLK	89 %
KCH	75 %
KNO	87 %
LON	67 %
MKA	90 %
NCH	84 %
OOG	82 %
ORT	87 %
PCH	82 %
URK	79 %
URO	88 %

Table 12: The utilization of each surgical department of 2015

VUmc has decided to use definition Eqn. 6, because they wanted to know the real time used, that is without turnover time. Further they only considered IOT. However, we do not think this definition is accurate. First, OOT (*outside operating time*) is still OR-time used. It can be time that was needed due to wrong scheduling, which chould be 'penalized' with less time the next year. Moreover, OOT could also have occurred due too little OR-time available for the type of surgery durations a department has. Therefore, excluding the overtime and OOT surgeries will give a lower time used than actually was used. Secondly, turnover time should also be included for the following reason. The time allocated to each surgical department is divided into many sessions in the blueprint (see Section 2.5.1). On these sessions, surgeries can be scheduled including turnover time (such as in Figure 5). Turnover time is always needed between two surgeries, but surgical departments should schedule in such a way that the turnover time is as low as possible. Therefore, a better definition for VUmc to use would be:

$$b_C = \frac{\sum_{m=1}^{j} \left(\sum_{n=1}^{i} d_{n,m} + (i-1)t \right)}{A \cdot co} \cdot 100\% \tag{7}$$

$$= \frac{R_t}{A \cdot co} \cdot 100\%. \tag{8}$$

In the model we have composed, we are looking at various lengths of S. We have calculated the utilization per surgical department, including turnover time and overtime for the reasons we have given above. For the explanation of the model see Section 6. We have simulated $\sum_{n=1}^{i} d_n + (i-1)t$ (with t=20), for different surgery combinations.
Thus, the definition we will use in this research is:

$$b = \frac{\sum_{n=1}^{i} d_n + (i-1)t}{S} \cdot 100\%. \tag{9}$$

Example 4.1. Suppose we have a session length $S = 465$ minutes, and three surgeries are executed on this session. These surgeries have taken $d_1 = 70$, $d_2 = 130$, and $d_3 = 200$. Recall, $t = 20$. Thus, $\sum_{n=1}^{i} d_n + (i-1)t = 70 + 130 + 200 + (3-1)t = 422$. The utilization is:

$$b = \frac{422}{465} \cdot 100\% = 91\%.$$

Note that in this case the OR utilization is the same as the utilization of a surgical department, because we consider only one session.

5 Data modeling

In Section 3, we have analyzed the data of the surgeries conducted in 2015. In this chapter we classify the surgery data in two different ways, as explained in Section 3.2. Recall, first with a medical point of view, and secondly merely by surgery durations .

5.1 Types of surgeries - Medically

As a first attempt to cluster surgeries, we have taken a medical point of view. By this, we mean that we cluster surgeries with the same type of medical procedures. Strum [17], and Oostrum [18] have done research on the best way to cluster surgeries by procedural terminology.
After determining the different types of surgeries for a surgical department, we can study the frequency of these types, analyze their average durations, and the differences in surgery durations within the types. By clustering surgeries instead of looking at them separately, we can predict more accurately and easily how often specific types of surgeries occur. We want to know this to determine the best session length, because those will depend on the mixture of surgery types.

Recall that each surgery has its own surgery number, and each surgery consists of one or more operations. Each operation has a code, containing six numbers and optional one letter at the end. The order of the operation codes does not matter. Therefore, we can put them in increasing order to make it easy to see which surgeries are the same, i.e. consist of the same operations.
Here, we have performed an analysis on all elective surgeries conducted in 2015. We have clustered the surgeries in the following way, using Microsoft Excel:

Algorithm 5.1.

1. Take the operation code without letters.
2. Order the codes in increasing order.
3. Combine the ordered operation codes belonging to the same patient-surgery number combination.
4. Cluster surgeries based on the description in the operation codes.
5. Compare the surgery durations within a type.
6. When there are large differences between the surgery durations within a type, adjust the types and add a type when needed, and go back to step 5.
   Otherwise, go to step 7.
7. Make a box-plot of the surgery durations within all types determined.

Details on our clustering method have been elaborated in the following two sections by looking at the surgical department Neurosurgery (NCH) and Urology (URO). We have chosen these two departments, since NCH already has been assigned some longer sessions than the regular ones of 465 minutes and URO has not, in spite of the fact that they claim to need it. Therefore, it could be interesting to compare these two departments.

5.1.1 Neurosurgery

In this section we apply the method described to department NCH.

Surgical department NCH executed 449 elective surgeries in 2015. Using the two data sets described in Section 3, we know all operations belonging to those surgeries. These 449 surgeries consisted of 741 operations in total.

First, we have combined the operation codes associated with the same patient-surgery number combination. This way, we have obtained 198 unique operation code combinations. This is still too much to handle. Therefore, we will cluster surgeries. First, we have looked at how many unique surgeries remain after removing all letters in the operation codes.

Example 5.2. Consider the following surgeries, corresponding to different operations:

Surgical department	Combination of operations	Occurrence
NCH	330114I_330159A	1
NCH	330159A_330114I	1
NCH	330114I_330165F	3
NCH	330165F_330144I	2
NCH	330165E_330144I	27
NCH	330114I_330165E	13
	Different types:	6

Excluding all the letters, we get:

Surgical department	Combination of operations	Occurrence
NCH	330114_330159	1
NCH	330159_330114	1
NCH	330114_330165	16
NCH	330165_330144	29
	Different types:	4

This way we are left with four surgery types, instead of six.

Applying the same procedure to all operations, we have 147 different combinations.
The surgery unit leader of VUmc has told us that the order of documenting operation codes does not matter. Hence, we will put the operation codes in increasing order, so that we always have the same order.

Example 5.3. Continuing the previous example, we get:

Surgical department	Combination of operations	Occurrence
NCH	330114_330159	2
NCH	330114_330165	45
	Different types:	2

So compared with the previous situation, we do not have four different types, but only two.

Doing this for all operations, we get 127 unique operation code combinations.

Further, the clustering is based on the description of the operation codes. It is preferable to have little deviation between the surgeries within a surgery type, otherwise, it will be difficult to make predictions. After clustering the first time, we had eight different types of surgeries. We thought the deviation in surgery times within three types were too high. Therefore, we ended up with the following eleven types (the letters below are randomly chosen from what we have read in the Dutch operation descriptions):

1. H: Surgeries concerning the skin.
2. HS: Brain surgeries.
3. PZ: Short surgeries concerning the peripheral nervous system.
4. PZ2: Long surgeries concerning the peripheral nervous system.
5. R: Short surgeries concerning the spinal cord.
6. RT: Long surgeries concerning the spinal cord.
7. S: Surgeries concerning the skull.
8. TO: Tumor surgery.

9. V: Surgeries concerning the heart.
10. W1: Short surgeries concerning something with the vertebral column.
11. W2: Long surgeries concerning something with the vertebral column.

The data of 2015 then yield the following table:

Type	Occurrence	Different surgeries in type	Average surgery duration	Standard deviation
H	6	5	166	133
HS	32	10	120	33
PZ	32	11	111	48
PZ2	21	13	247	97
R	44	10	125	95
RT	36	10	314	91
S	34	8	242	135
TO	182	31	385	123
V	42	11	113	33
W1	8	7	156	21
W2	12	11	333	93
Total	449	127	265	

Table 13: The surgery types of Neurosurgery

Figure 6 is a visualization of the occurrence of the types we have made, and gives the average duration of a surgery in those types.

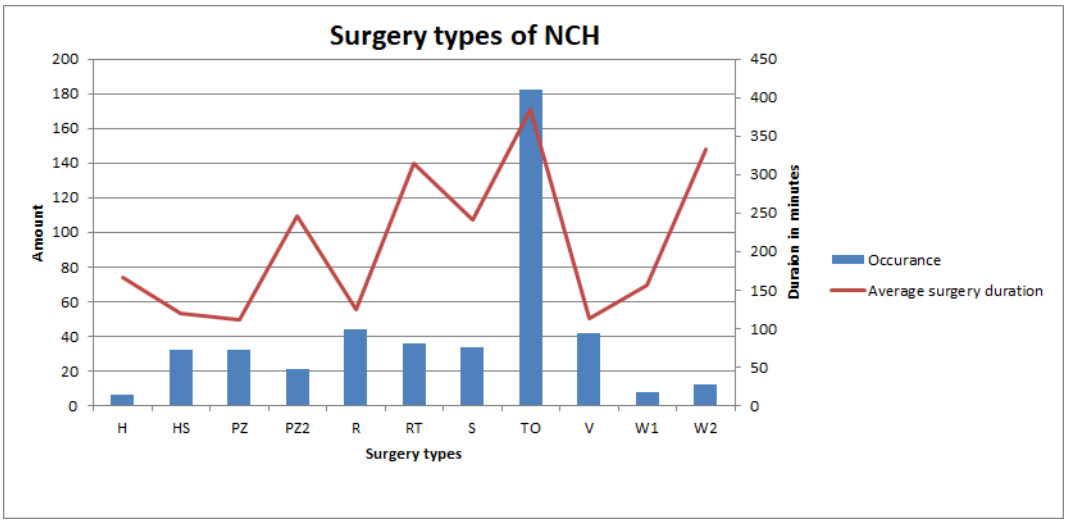


Figure 6: The occurrence of the surgery types of Neurosurgery with their average duration.

We see that the type 'TO' is the most frequent one. So, the probability is highest that we have to schedule a surgery belonging to that type. Therefore, we expect that the best session length will depend on the duration of the surgeries of type 'TO'.

The next figure, Figure 7, shows the deviation in surgery duration within each type of NCH.

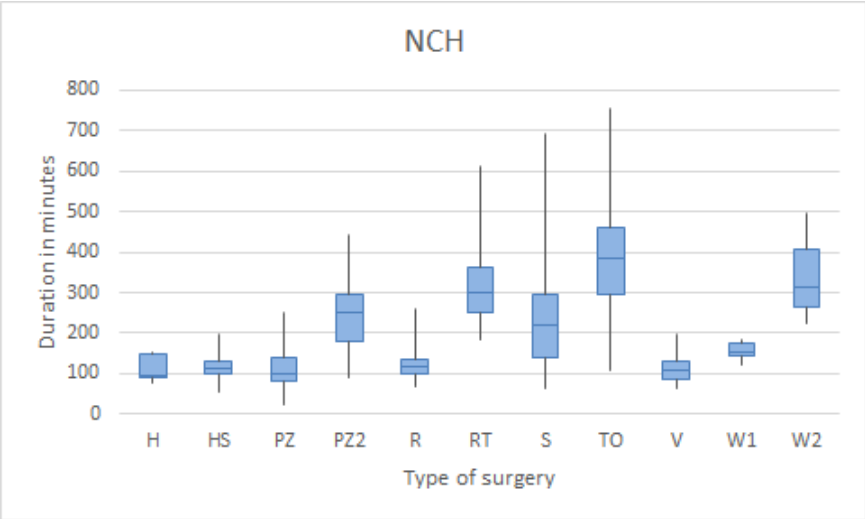


Figure 7: Box-plot of the surgery durations within each surgery type of Neurosurgery.

Earlier, we have seen that type 'TO' is the most frequent type. However, it is interesting to note that for this type we have the largest difference between the minimal and maximal surgery duration. This might be a problem in the simulations concerning the determination of the best session length. First, we will look at another department.

5.1.2 Urology

Department URO performed 282 surgeries in 2015. After combining the operation codes including the letters and not looking at the order, 121 unique surgeries remained. After deleting the additional letters from the operation codes, we ended up with 116 unique surgeries. The last step, ordering the operation codes in increasing order gave us 111 unique surgeries.

After clustering the surgeries, we obtained nine types. Again, the type names (letters below) are randomly chosen from what we have read in the Dutch operation descriptions:

1. B: Brain surgery.
2. C : These surgeries were hard to cluster with other types, but did fit together by their duration.
3. LS : Surgeries concerning the lymphatic system.
4. M : Surgeries concerning man.
5. NN: Kidney surgery.
6. U: Short surgeries concerning the ureter.
7. U2: Long surgeries concerning the ureter.
8. UR: Short surgeries concerning the urethra.
9. UR2: Long surgeries concerning the urethra.

Given the data of 2015, the following table yields:

Type	Occurrence	Different surgeries in type	Average surgery duration	Standard deviation
B	59	59	82	40
C	4	4	456	44
LS	72	72	281	89
M	65	65	161	58
NN	13	13	219	70
U	12	12	103	39
U2	21	21	343	77
UR	28	28	168	40
UR2	8	8	70	32
Total	282	191	195	

Table 14: The surgery types of Urology

Figure 8 gives a visual of the occurrence of the types we have composed, and gives the average duration of an surgery in those types. We see that three types ('B', 'LS', and 'M') have a much higher occurrence then the other types. So, the best session length will most likely depend on the surgery durations of those types.

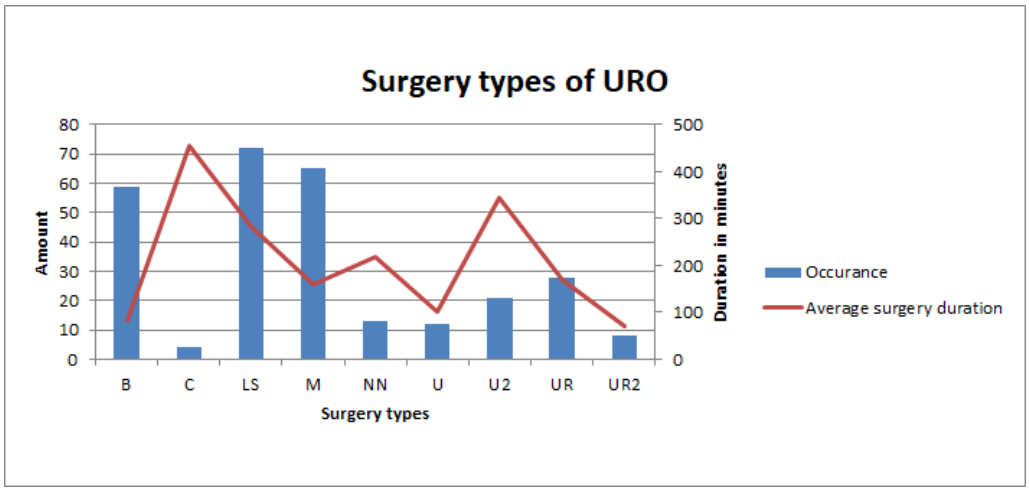


Figure 8: The occurrence of the surgery types of Urology with their average duration.

The next Figure 9 shows the deviation in surgery duration within each type of NCH.

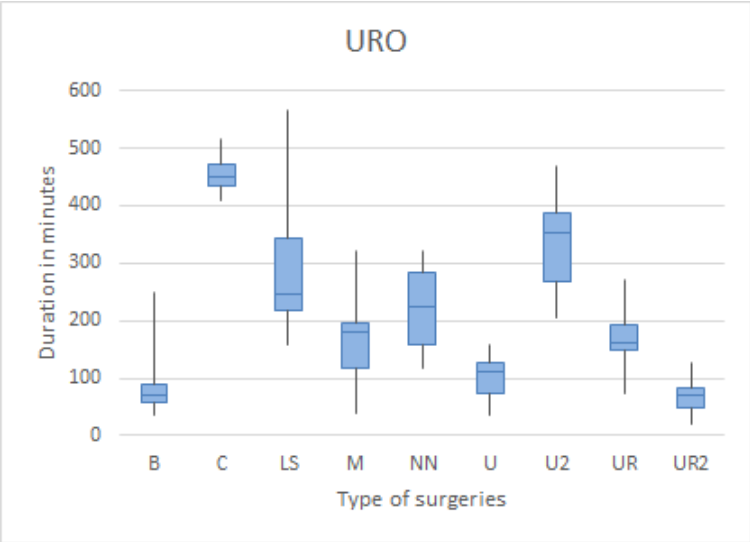


Figure 9: Box-plot of the surgery durations within each surgery type of Urology.

In Table 14 and Figure 8, we have seen that types 'B', 'LS', and 'M' have the highest occurrences. In the above figure we see that the minimal surgery duration of those types to the maximal duration, covers almost all surgery durations of URO. Therefore, we can't make any conclusions about the best session length just yet.

5.1.3 Conclusions

After determining the surgeries types of NCH and URO, we decided we want to design a method to find surgery types for all other surgical departments quicker than doing it by hand as we did above. Given the types for each department, we want to determine the best session length for each department based on specific combinations of types.

Example 5.4. Suppose we have a particular surgical department with two types of surgeries. One type has an average duration of seven hours and the other one of three hours. We also know that the surgeries of three hours occur three times more often than the seven hour surgeries. Now, suppose the department has only sessions of eight hour length, so it is not possible to schedule one seven hour surgery and one three hour surgery on the same day without overtime. Also, when only one seven hours surgery is planned, undertime will occur. Undertime will give a lower OR utilization, and overtime usually comes along with extra costs.
Assume we give this department sessions of seven hours and of nine hours. Then, we will not have overtime or undertime (due to planning) when a seven hour surgery is scheduled on a seven hour session, or when three three hour surgeries are scheduled in the nine hour session.

As mentioned in Section 3.2, management has decided not to proceed with clustering surgeries from a medical point of view. The main reason is that in this setup, the session lengths are based on the types we have composed. When we base session lengths as in example 5.4, we implicitly are designing planning rules on what surgery types are medically speaking good to be scheduled together. However, some other projects of VUmc are focused on this subject. Also, we do not want to make planning rules, because in this research we intend to make a new blueprint, not a new surgery schedule.
The idea of clustering from a medical point of view was to get types with relatively low standard deviation. However, we do not immediately see a big advantage for our session length analysis. Clustering this way might be helpful when optimizing scheduling surgeries per session. This is inline with managements request.

In the next section, we have clustered surgeries merely by their durations. However, clustering surgeries from a medical point of view is more meaningful than just clustering by surgery duration. Fortunately, the mean idea remains the same we only have to make some more assumptions.

5.2 Classification by surgery duration

The other way of clustering we mentioned is by surgery duration. We will not analyze what kind of surgeries were executed, and what reason causes the deviation in surgery times, but we are going to look at the frequency of surgery durations.

Here, we used all elective surgeries performed in 2015. However, we have made two alterations. First, Neurosurgery executed one surgery with a duration of 1118 minutes. The next longest surgery, was 754 minutes. We excluded the surgery of 1118 because it only occurred once. Therefore, it is not highly likely to occur in the next year. When it does occur, some additional OR-time or staff could be made available when necessary. So, we only looked at surgeries of 754 minutes and lower.
Similarly, the department KCH had one surgery of 1430 minutes and one of 1075 minutes. All other surgeries had a duration of at most 835, with only small differences in surgery times. We excluded the two surgeries of 1430 and 1075 minutes, for the same reason as we excluded one surgery for NCH.

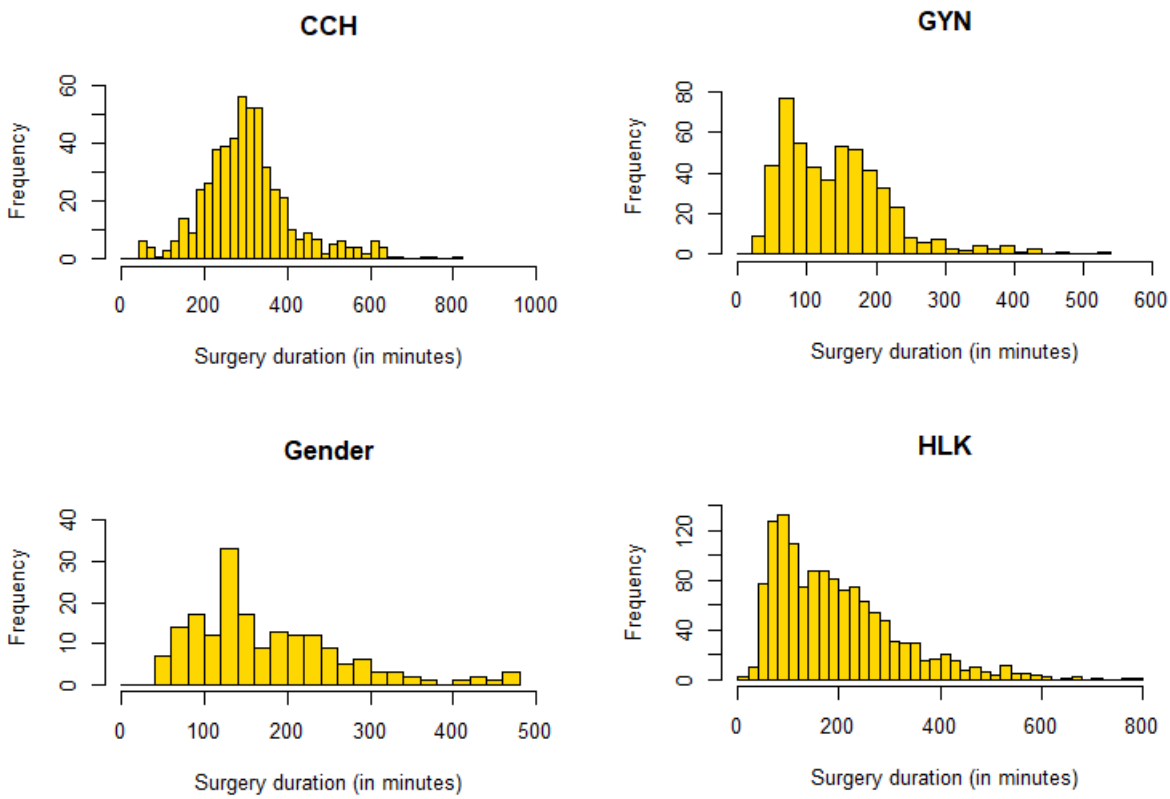
We have used the statistical software package R to perform the biggest part left of this research.

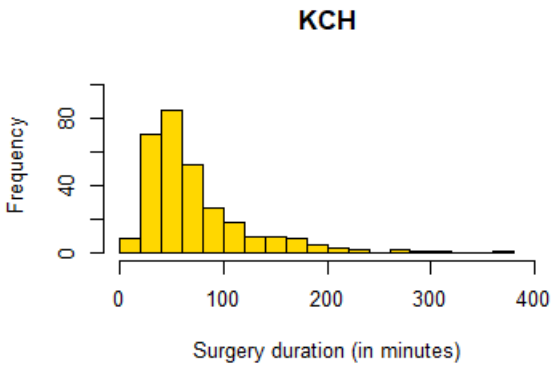
5.2.1 Frequency of surgery durations

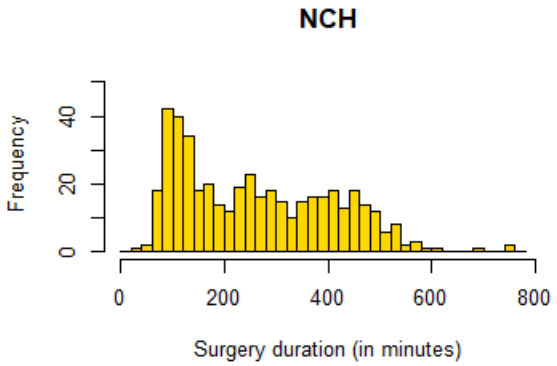
We have uploaded a cvs-file with our surgery data to R (which is confidential), and for each surgical department we made a frequency plot (histogram) of the realized surgery durations in 2015. The histograms are based on 20 minutes intervals, since we have a constant turnover time of 20 minutes. Choosing the intervals the same seemed to be a logical choice to prevent confusion. The set of intervals is:

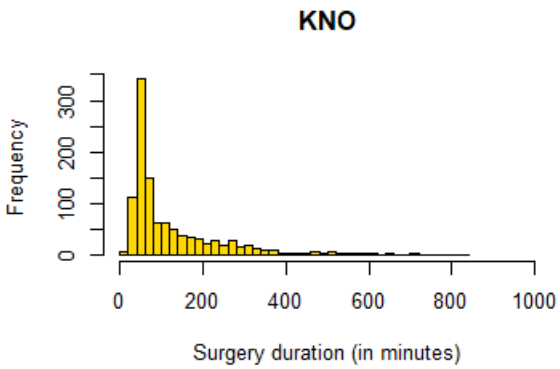
$$X = \{P20, P = 0, 1, 2, ..., p\} \text{ with } p = \left\lceil \frac{max\ duration}{20} \right\rceil . \tag{10}$$

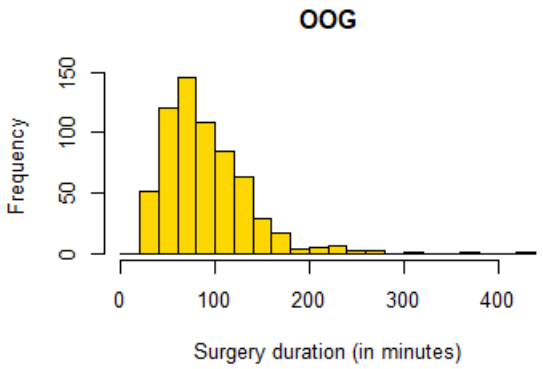
The histograms of all surgical departments are given below.
The R-code can be found in Appendix D.1 and D.2.

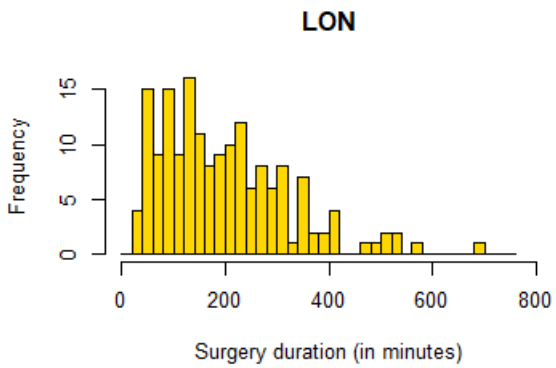


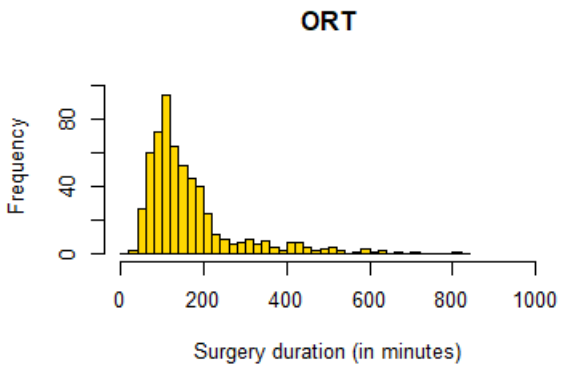

KCH
Frequency
0
40
80
0
100
200
300
400
Surgery duration (in minutes)

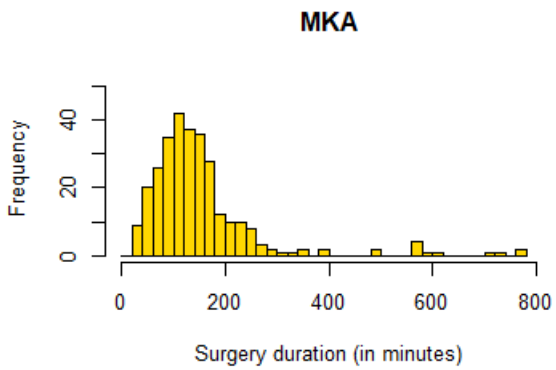

NCH
Frequency
0
20
40
0
200
400
600
800
Surgery duration (in minutes)

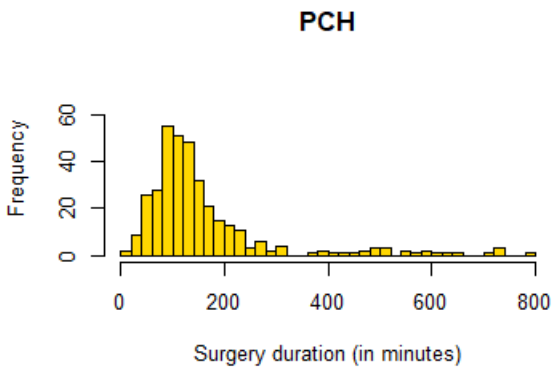

KNO
Frequency
0
100
200
300
0
200
400
600
800
1000
Surgery duration (in minutes)

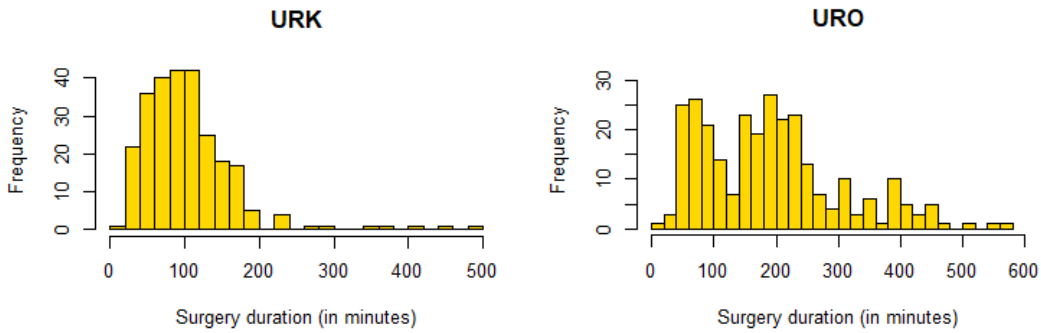

OOG
Frequency
0
50
100
150
0
100
200
300
400
Surgery duration (in minutes)


LON
Frequency
0
5
10
15
0
200
400
600
800
Surgery duration (in minutes)


ORT
Frequency
0
40
80
0
200
400
600
800
1000
Surgery duration (in minutes)


MKA
Frequency
0
20
40
0
200
400
600
800
Surgery duration (in minutes)


PCH
Frequency
0
20
40
60
0
200
400
600
800
Surgery duration (in minutes)



Some observations on the histograms of the departments:

- For most of the surgical departments the range of surgery durations is rather small, i.e. one top and a long right skewed tail. However, we see that some departments have two (GYN) of three (NCH and URO) tops.
- The occurrence of surgeries with a very high duration, more than 300 minutes, has a low probability. This might be due to the maximum session length assigned.
- We can conclude from the histograms that surgical departments KCH, OOG, ORT, and URK mostly have short surgeries.

5.2.2 Probability distributions

As a next step, we want to fit the histograms to a probability distribution in order to make predictions on the best session length. In order to determine the best session lengths, we do not want to use the actual data, but rather a probability function that is a good fit on the data. Therefore, the results will be statistical more accurate.
First, we plotted a density graph using a R-function on all histograms to get an insight on how the probability distributions look like. In [22] on page 32, it is mentioned that a log-normal distribution is often a good distribution to model surgery durations. On the other hand, some of the plots seem to be better fitted by a gamma distribution. Therefore, we have also tried to fit that distribution to the data of each department to find the best fit.

Using the '*fitdistr*'-function of the MASS package of R, we get the parameters corresponding to the data of the surgical departments for different probability distributions. Appendix C.3 states a report of our attempts and findings for the different probability distribution fits for each of the departments. The corresponding R-code can be found in Appendix D.2.
Below we give some of the main observations.

- Gender: The density plot of R follows a gamma distribution with parameters $k \approx 4.19$, and $\theta \approx 0.02$. However, the actual data has a larger top.
- GYN: Most likely, a bimodal distribution is needed.
- HLK: The density plot of R follows a log-normal distribution with parameters $\mu \approx 5.07$, and $\sigma \approx 0.64$.
- NCH: Most likely, a trimodal distribution is needed.
- OOG: The density plot of R follows a gamma distribution with parameters $k \approx 4.79$, and $\theta \approx 0.05$.
- URK: The density plot of R follows a gamma distribution with parameters $k \approx 3.45$, and $\theta \approx 0.03$.
- URO: Most likely, a trimodal distribution is needed.

For the other surgical departments, it was hard to find a distribution that fitted the surgery duration data properly. We tried the log-normal and gamma distribution with different parameters on the data. However, this did not yield good results. As a result we have decided to use the empirical probability distributions for all departments.

The empirical probabilities of a specific surgery duration have been calculated as follows. The empirical probability is equal to the ratio of the frequency of a surgery duration and the total number of surgeries executed by the surgical department. Thus the probability that a surgery has duration d_l $(l = 0, 1, ..., p-1)$ is:

$$\mathbb{P}(D = d_l) = \frac{freq(d_l)}{\sum_{n=1}^{p} freq(d_n)}, \tag{11}$$

with $freq(d_l)$ is the number of surgery durations with values in $[0, 20]_{l=0}$ and $(l20, 20(l+1)]_{l\geq 1}$.
We will calculate the probabilities on a surgery duration using the middles of the bars of the histograms. Let

$$Y = \{10 + 20l, l = 0, 1, ..., p-1\}$$

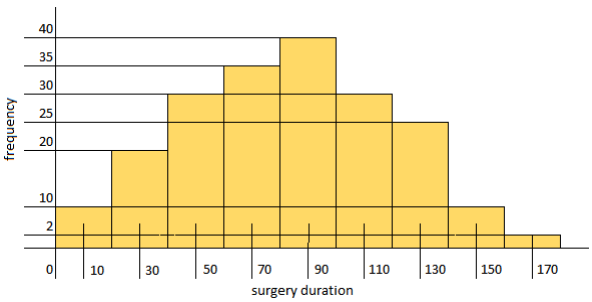
be the potential surgery duration values. Then, with $d_l = 10 + l20$:

$$\mathbb{P}(D = d_l) = \frac{\sum_{d_n \in (l20,...,l20+20]} d_n}{\sum_{n=1}^{p} freq(d_n)}. \tag{12}$$

Thus:

$$\mathbb{P}(D = d_l) = \frac{\sum_{d_n \in \{d-9, d-8, ..., d+9, d+10\}} d_n}{\sum_{n=1}^{p} freq(d_n)}. \tag{13}$$

Example 5.5. Suppose we have a surgical department with surgery durations given in the histogram on the right. Then the total amount of surgeries performed by this department is



$$\begin{aligned}\sum_{n=1}^{p} freq(d_n) &= 10 + 20 + 30 + 35 + 40 + 30 + 20 + 10 + 2 \\ &= 197\end{aligned}$$

So, the probabilities of a surgery with duration $d = 10$ or $d = 30$ are equal to

$$\mathbb{P}(D = 10) = \frac{10}{197}, \quad \mathbb{P}(D = 30) = \frac{20}{197},$$

etc.

When determining the optimal session length for each surgical department, we do not use the actual data. We want statistically accurate probabilities, so we have drawn 10,000 times from the empirical probability distributions. We have used the function *set.seed()* of R, so we always work with the same (reproducible) probabilities. In Section 6, we discuss this further.

The next chapters will answer the question whether variable session lengths per surgical department (all their own best session length) will contribute to a better utilization.

6 Mathematical modeling

So far, we have thoroughly analyzed the current situation, and we have gotten an understanding of the surgery durations within each surgical department. Furthermore, we have given the optimization criteria in Section 4.2. The next step is to build a mathematical model to determine the best session length for each department, with which we can conclude whether those session lengths result in a better utilization.

In this chapter we explain how the model is composed and how it works. Also, the assumptions that have been made in the model are given and explained. The model will be made and executed using R.

6.1 What do want to model

To be able to determine the best session length for each department, we need to start with calculating the expected over- and undertime for different session lengths. We will not study all possible session lengths, because certain lengths will not be realistic due to being either too short or too long. In Section 4.2 we have explained what the possible session lengths are, namely $300 \leq S \leq 600$ minutes for all departments except for KNO and PCH, who have $300 \leq S \leq 720$. In the calculations in R, we take $290 \leq S \leq 610$ and $290 \leq S \leq 710$, due to the way have we set up the empirical probability distributions (cf. Section 5.2.2).

First, we have considered the case of one surgery per session, where the surgery duration is drawn from the empirical probability distribution of the surgical department in question. Then we calculate the expected over- and undertime for different session lengths. This will be explained in the next section. Based on the expected over- and undertimes for only one surgery per session, we can determine what the best session length is for each of the criteria in Section 4.2. Also, the utilization for all concerned session lengths needs to be calculated.
Next, we have considered the case of two surgeries per session. The total duration of two surgeries including turnover time, is equal to the convolution of the empirical probability distributions of those two surgeries plus the constant turnover time of 20 minutes. Again, the expected overtime, undertime, and utilization are calculated for the different session lengths. The same has been done for three and more surgeries per session. The number of surgeries a surgical department has executed per session has been given in Section 3.4.

We will obtain a list of expected over- and undertime for different session lengths when scheduling different amounts of surgeries per session. With that list we can determine the best session length for each surgical department, whatever the amount of surgeries executed per session is.

6.2 How do we calculated utilization, over- and undertime

Recall the definitions of over- and undertime given in Section 1.4.

Suppose we have a session of length S, and schedule one surgery. Assume the surgery ends up having a duration d. Then the undertime u and the overtime o are:

$$u = (S - d)^+ \tag{14}$$
$$o = (d - S)^+ \tag{15}$$

Using the empirical probability distribution we know the probability that a surgery took d minutes to complete. So, the expected overtime $\mathbb{E}[o]$ and the expected undertime $\mathbb{E}[u]$ are (for one surgery per session):

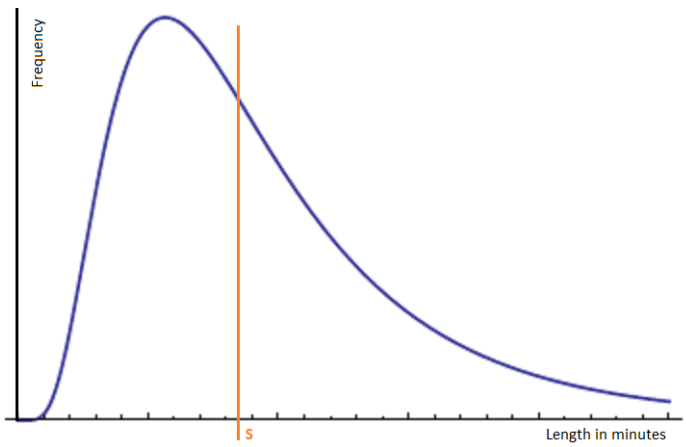


Figure 10: Example empirical probability distribution

$$\mathbb{E}[u] = \sum_{d<S} (S-d)\mathbb{P}(D=d), \tag{16}$$

$$\mathbb{E}[o] = \sum_{d>S} (d-S)\mathbb{P}(D=d). \tag{17}$$

Logically, when $S = d$ there is no under- or overtime.

When more and more surgeries are scheduled per session, for the convolution of the empirical probability distributions (including turnover time) the following theorem will hold.

Theorem 6.1. ***Central Limit Theorem*** [2] [3]
Let $\boldsymbol{X} = (X_1, X_2, ...)$ *be a sequence of independent, identically real-valued random distributed variables having mean* μ *and variance* σ^2 *with common distribution function* F*. Let*

$$S_n = \sum_{i=1}^{n} X_i.$$

Then

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{S_n}{\sigma\sqrt{n}}\right) = \Phi(x), \quad -\infty < x < \infty$$

where Φ *is the standard normal distribution function.*

Although, we have not succeeded to fit probability distributions to the surgery data, this theorem shows that for many surgeries per session, we can use a normal distribution. However, we will keep using the empirical probability distributions. When two surgeries are performed per session, the probability that the sum of those two surgeries durations is $d(2)$ minutes follows of the **convolution** of the empirical probability distributions of those surgeries plus 20 minutes turnover time.
Suppose we have empirical probability distribution D of a department C. We want to determine the probability distribution in case of two surgeries on the session. The duration of the first surgery is independent of the duration of the second surgery. Therefore:

$$\mathbb{P}(D(2) = d(2)) = \left(\sum_{d(2)=-\infty}^{\infty} \mathbb{P}(D=d)\mathbb{P}(D=d(2)-20-d) \right). \tag{18}$$

We will always calculate the new empirical probability distribution first, and use Eqns. 16 and 17 to easily calculate the expected over- and undertime for every combinations of surgeries we want to consider.

As mentioned in the beginning of this chapter, we have used R to do the calculations. We have coded the following functions to calculate the expected overtime o, undertime u, and utilization b.

[2] *Mathematical Statistics and Data Analysis.* John A. Rice, Third edition. Thomson Brooks/Cole. Chapter 5, page 184.
[3] http://www.math.uah.edu/stat/sample/CLT.html

```
# k are the empirical probabilities of surgery durations in
  #the distribution that is concerned
# d are the durations, the middles of the histogram's bars
u<-function(k,d){
  m<-matrix(nrow=length(s),ncol=length(d))
  for(i in 1:length(s)){
    for(j in 1:length(d)){
      if(d[j]<s[i]){m[i,j]<-(s[i]-d[j])}
    }
  }
  uu<-sapply(1:nrow(m),function(l) sum(m[l,]*k[2],na.rm=TRUE))
  uu
}
o<-function(k,d){
  m<-matrix(nrow=length(s),ncol=length(d))
  for(i in 1:length(s)){
    for(j in 1:length(d)){
      if(d[j]>s[i]){m[i,j]<-(d[j]-s[i])}
    }
  }
  oo<-sapply(1:nrow(m),function(l) sum(m[l,]*k[2],na.rm=TRUE))
  oo
}
b<-function(k,d){
  m<-matrix(nrow=length(s),ncol=length(d))
  for(i in 1:length(s)){
    for(j in 1:length(d)){
      if(d[j]<s[i]){m[i,j]<-(s[i]-d[j])}
      else {m[i,j]<-d[j]}
    }
  }
  bb<-sapply(1:nrow(m),function(l) sum(m[l,]*k[2])/s[l]*100)
  bb
}
```

Figure 11: Function in R to calculate the expected overtime o, undertime u, and utilization b

The input of the function is the empirical probabilities, and the surgery durations we want to consider. The session length S satisfies either $290 \leq S \leq 610$ or $290 \leq S \leq 710$. First, we fill a $length(S_m) \times length(d_n)$ matrix with $(S-d)\ \forall\ d < S$, which are all possible cases of undertime. The other elements of the matrices stay empty. Each row of the matrix represents a different session length, and each column a surgery duration that is concerned. When we multiply each element in the row with the empirical distributions represented as a column vector, we obtain the expected undertime when we have that session length. The same holds for the overtime, with the matrix filled with $(d-S)\ \forall\ d > S$. The functions for the expected utilization combines these two, and the definition given in Section 4.3 is used, see Eqn. 9.

6.3 Mathematical model: Types of duration

Recall $300 \leq S \leq 600$ (or 720), therefore we have considered all possible session lengths between 290 and 610 (resp. 710) minutes due to how the empirical probabilities are calculated, using 20 minutes intervals. Then, we can use Eqn. 13. Some surgeries or combinations of surgeries will be very short or very long. So, we don't want to look at these combinations because they have a negative influence on the expected undertime or overtime. Therefore, next to taking the requirements regarding the session length, we have divided the surgery durations of each surgical department into groups, further called '*surgery duration types*' or '*surgery duration groups*'. Making surgery duration groups, it has become easier to determine which combinations we do and do not want to schedule together.

We have designed the following algorithm:

Algorithm 6.2.

1. Consider surgical department C.
2. Let k be the amount of surgery duration groups we want to have, and
3. $Q = \{q_1, q_2, ..., q_{k-1}\}$ the set of boundaries, with $0 < q_1 < q_2 < ... < q_{k-1}$.
4. Then, we have the groups
$$V = \{V_1, V_2, ..., V_k\}, \text{ with}$$
$$V_1 = \{v \in Y | 0 \leq v \leq q_1\},\ V_{1<l<k} = \{v \in Y | q_{l-1} < v \leq q_l\},\ V_k = \{v \in Y | v > q_{k-1}\}.$$
5. Each group has its own empirical probability distribution. Draw 10,000 times, then
6. the probability we draw a surgery of group V_g $(g = 1, .., k)$ is,
$$\mathbb{P}(V_g) = \mathbb{P}(D_g \leq q_g), \text{ where} \tag{19}$$
$$\mathbb{P}(D_g = d) = \frac{\mathbb{P}(D = d)}{\mathbb{P}(D \in V_g)},\ d \in V_g. \tag{20}$$
7. Table 3.4.1 gives the potential number of surgeries to schedule per session for department C, say n. Where n is a list of numbers not necessarily starting at 1.
8. Use $\sum_n \binom{k+n-1}{k-1}$, to determine the how many combinations of V are possible for each number of surgeries we consider (from the previous step). [*]
9. A combination of surgery groups of V is only feasible, when their mean surgery duration is more than 300 minutes, and less than 600 minutes (except for PCH, and KNO: less than 720 minutes). This is a list $H_n = \{H_{1,n}, H_{2,n}, .., H_{h,n}\}$ with $H_{r,n}$ $(r = 1, .., h)$ a collection of n numbers from $\{1, ..., k\}$.
10. Use Eqn. 18 to determine the empirical probability distribution for each group combination for $n > 1$.
11. Again, draw 10,000 times of these new empirical probability distributions, and use Eqn. 13 to determine the new empirical probabilities.
12. Re-normalize the probabilities $P(H_{r,n})\ \forall\ r, n$ as the probability of the group combination conditioned on the probability of all allowed group combinations (cf. Examples 6.3, and 6.4).
13. Determine the expected overtime, undertime, and utilization with the R-code given in Figure 11.
14. Then, the expected overtime, undertime, and utilization for each n is:
$$\mathbb{E}_n[o] = \sum_r \mathbb{P}(H_{r,n}) \cdot \mathbb{E}_{r,n}[o],$$
$$\mathbb{E}_n[u] = \sum_r \mathbb{P}(H_{r,n}) \cdot \mathbb{E}_{r,n}[u],$$
$$\mathbb{E}_n[b] = \sum_r \mathbb{P}(H_{r,n}) \cdot \mathbb{E}_{r,n}[b].$$
15. The expected overtime, undertime, and utilization for C are:
$$\mathbb{E}[o] = \sum_n \mathbb{P}(n)\mathbb{E}_n[o],$$
$$\mathbb{E}[u] = \sum_n \mathbb{P}(n)\mathbb{E}_n[u],$$
$$\mathbb{E}[b] = \sum_n \mathbb{P}(n)\mathbb{E}_n[b].$$

In Section 3.4 we have given how many surgeries each surgical department scheduled per session in 2015 (Step 7 of Algorithm 6.3), and we also gave the probability of the number of surgeries per session ($\mathbb{P}(n)$ in Step 15 of Algorithm 6.3). We have calculated the mean surgery duration of each combination using statistical software package R. Also, the other calculations done in algorithm above, are executed in R.

[*] Remark, in step 8 of the algorithm we use $\binom{k+n-1}{k-1}$ to determine the amount of combinations of surgery groups for different numbers of surgeries per session. Since [7] gave that shorter/easier surgeries should be scheduled before the longer ones, we assume the order of the constituting surgery durations to be fixed. Therefore, we have decided not to look at the order of the surgeries, assuming that VUmc always schedules surgeries of short durations before surgeries with a longer duration.

We will only consider $k = 2$ or $k = 3$ in this research.
An advantage of this approach of dividing the surgery durations per department into groups, is that we have excluded some of the main non-realistic surgery combinations. We have also simplified the model. However, it will be easier to determine the best session length for each surgical department, whatever amount of surgeries is scheduled.
There are also some limitations and disadvantages on making the groups. First, we have assumed that it is possible to make surgery types according to their durations. However, in practice this will not always be easy or logical. Secondly, we have decided to exclude combinations of groups when the mean duration is less than 300 or more than 600 minutes (720 minutes for KNO and PCH). However, our method to divide the surgery durations intro groups and selecting certain groups combinations, allows to schedule surgeries in one session that would not be combined in practice, and vice versa.

6.4 Different scenario's

In the previous section we have discussed how to divide the surgery durations of a department into groups. We did not specify how to select $q_1, ..., q_{k-1}$, just yet. We restrict to $k = 2$, and $k = 3$, and we have chosen two scenarios for determining the group boundaries.

Scenario 1:
First, we have divided the maximal observed surgery duration by 2 or 3. Then, we have rounded the value to the next larger multiple of 20. This is the first boundary. When we want three groups, the second boundary is twice the first boundary. Thus, for example for HLK (with HLKduur the set of surgery durations of HLK), q_1, q_2 are (determined in R):

```
> gr <- (max(HLKduur)/3) #dividing the maximal surgery duration by 3
> q1 <- gr-gr%%20+20     #we need to have a multiple of 20 #first boundary
> q2 <- q1+q1            #second boundary
```

Scenario 2:
A more mathematical way of making the groups is to take $q_1 = mean - standard\ deviation$ and $q_2 = mean + standard\ deviation$ of the surgery durations. We have based this on [22], page 30. This works when we want to have three groups. When we want two groups, we will take $g_1 = mean\ surgery\ duration$ as the boundary between the groups.

The table below gives the mean, standard deviation (sd) and the two boundaries for each surgical department. We have used the code in Figure 12 to calculate the means and standard deviations.

	Mean	sd	mean-sd	mean+sd
CCH	306.76	110.12	196.64	416.88
Gender	175.05	90.53	84.52	265.58
GYN	144.53	78.68	65.85	223.21
HLK	193.67	121.23	72.45	314.90
KCH	73.69	51.77	21.92	125.46
KNO	125.54	122.85	2.69	248.40
LON	198.93	124.88	74.05	323.81
MKA	154.83	116.44	38.40	271.27
NCH	262.58	146.43	116.16	409.01
OOG	89.54	43.94	45.60	133.48
ORT	166.90	114.78	52.11	281.68
PCH	161.46	133.65	27.81	295.12
URK	104.25	62.36	41.89	166.61
URO	190.49	109.55	80.93	300.04

Table 15: The mean surgery duration, and the corresponding standard deviations.

```
#list of surgery durations of all surgical departments
LSD<-list(CCHduur,Genderduur,GYNduur,HLKduur,KCHduur,KNOduur,LONduur,MKAduur,NCHduur,
          OOGduur,ORTduur,PCHduur,URKduur,UROduur)
names(LSD)<-c("CCHduur","Genderduur","GYNduur","HLKduur", "KCHduur","KNOduur","LONduur",
              "MKAduur","NCHduur","OOGduur","ORTduur","PCHduur","URKduur","UROduur")

# to calculate the mean, and standard deviation of each surgical department
GDS<-sapply(names(LSD), function(C){
  set.seed(1234)
  E<-EmpiricalDistribution(LSD[[C]])
  r<-r(E)(10000)
  t(data.frame(mean(r),sd(r),mean(r)-sd(r),mean(r)+sd(r)))
})
GDS<-t(GDS)
GDS<-as.data.frame(GDS)
names(GDS)<-c("Mean","sd","mean-sd","mean+sd")
```

Figure 12: R-code to calculate the mean and standard deviation of each departments

A slight deviation in the mean surgery duration can be noticed between Table 9 and 15. We have based the empirical probability distributions on all elective data (Table 15), and in Table 9 we have only looked at the surgeries executed during the departments own sessions.

Appendix C.4, for each surgical department both scenarios (named 'types 1' and 'types 2') are given. The surgical departments NCH and CCH get two groups of surgery durations, and the other departments three. We have represented the groups by giving each group in the histograms a different colour, see Figure 13. We used the *cut()* function of R for this. We can see that in the second way of making the groups, the first group is much smaller. Therefore, we have different possible ways of scheduling surgeries per session which are allowed in both scenarios.

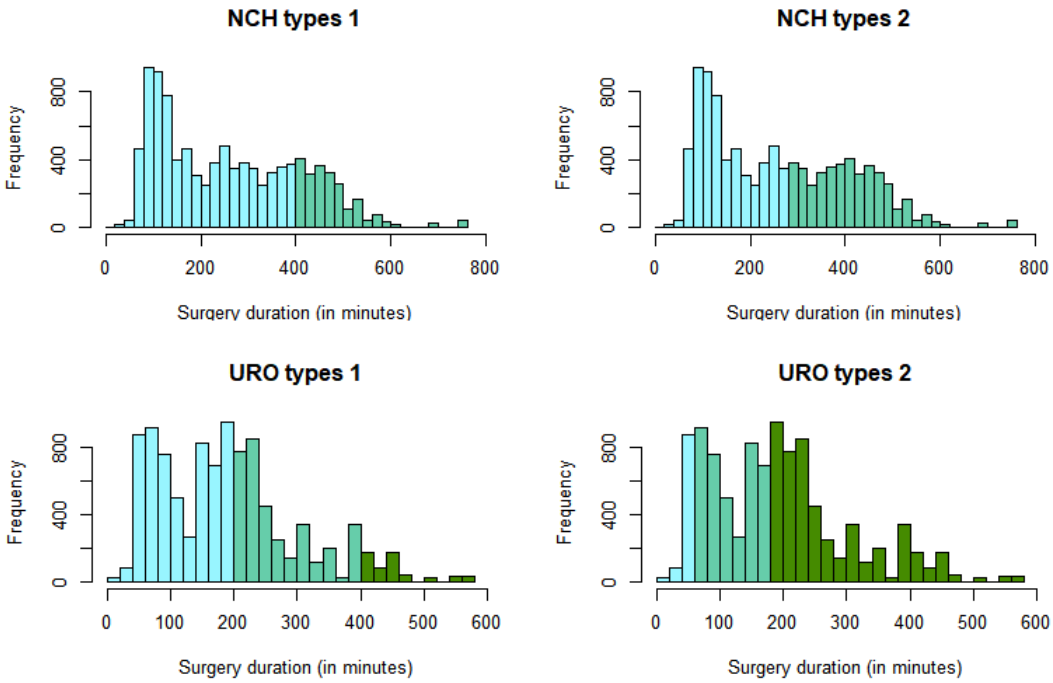


Figure 13: The groups (types 1 versus type 2) for NCH and URO

As mentioned in Example 6.3, each group has its own empirical probability distribution. The next figure shows for NCH (two groups), and URO (three groups) how we determined these in R. This code is usable for both scenario's (and all departments), we only have to specify how the boundaries are defined.

```
set.seed(1234)
#duration group 1 of NCH
NCHduur1<-NCHduur[NCHduur<=br] #br is the boundary
#duration group 2 of NCH
NCHduur2<-NCHuur[NCHduur>br]

# the empirical probability distributions of NCH
E<-EmpiricalDistribution(CCHduur)
#the empirical probability distribution for both groups of NCH
E1<-EmpiricalDistribution(CCHduur1)
E2<-EmpiricalDistribution(CCHduur2)


#duration group 1 of URO
UROduur1<-UROduur[UROduur<=br1] #br1 is boundary one
#duration group 2 of URO
UROduur2<-UROduur[UROduur<=br2&UROduur>br1]
#duration group 3 of URO
UROduur3<-UROduur[UROduur>br2] #br2 is boundary two


# the empirical probability distributions of URO
E<-EmpiricalDistribution(UROduur)
#the empirical probability distribution for all groups of URO
E1<-EmpiricalDistribution(UROduur1)
E2<-EmpiricalDistribution(UROduur2)
E3<-EmpiricalDistribution(UROduur3)
```

Figure 14: The R-code to determine the groups for NCH and URO

The next section gives the execution of Algorithm 6.3 on surgical departments CCH, and HLK, as an example. Here, we used scenario 1 for making the surgery duration groups.

6.4.1 Examples

We have given the results of execution of Algorithm 6.3 for departments CCH, and HLK. We have chosen to show the results of these two departments, because for CCH we have $k = 2$, and for HLK $k = 3$.

Example 6.3.
Cardio surgery (CCH) schedules one or two surgeries per session. Therefore, we can conclude that CCH has two types of surgeries, $k = 2$. One of medium long duration V_1, and one of long duration V_2. Then, we have $\binom{2+1-1}{2-1} + \binom{2+2-1}{2-1} = 5$ possible ways of combining the two surgery duration groups:

- One surgery:
  - One medium long surgery: $H_{1,1} = \{1\}$
  - One long surgery: $H_{2,1} = \{2\}$
- Two surgeries:
  - Two medium long surgeries: $H_{1,2} = \{1, 1\}$
  - One medium long surgery and one long surgery: $H_{2,2} = \{1, 2\}$
  - Two long surgeries: $H_{3,2} = \{2, 2\}$

The boundary of the two groups is:

```
> gr <- (max(CCHduur)/2)  #403
> q1 <- gr-gr%%20+20      #420
```

See Figure 15, the probability we have drawn a surgery of group V_1 respectively V_2 is

$$\mathbb{P}(V_1) = \mathbb{P}(D \leq q_1) = 0.8865, \quad \mathbb{P}(V_2) = \mathbb{P}(D > q_1) = 0.1135,$$

with the empirical probabilities calculates as in Section 5.2.2.

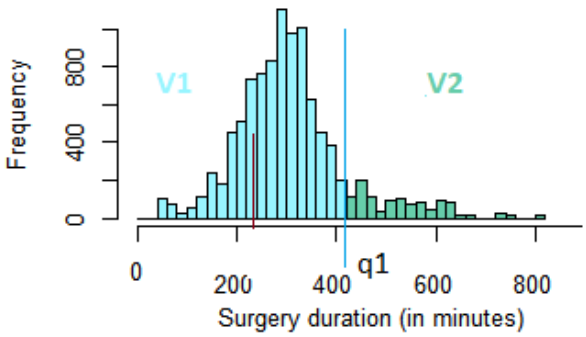


Figure 15: Example: Two groups of surgery durations

We have also 'divided' the empirical probability distribution of CCH into two groups. Then we get two new empirical probability distributions. We have 10,000 times from these 'new' distributions, and calculated the probabilities on a surgery within each of groups similar to Section 5.2.2. Hence, we will not use Eqn.20.

For each of the combinations mentioned earlier, we have calculated the mean durations. When the mean duration of a surgery group or combinations of groups is below 300 minutes of higher than 600 minutes, we will not consider the group or combination of groups as a possible way of planning surgeries. So, we might exclude some of the possible combinations. We have determined the mean surgery durations based on the emperical probability distributions. Therefore, we will still consider some surgery combinations longer than 600 or shorter than 300 minutes. Thus, our calculations on the expected over- and undertimes will not always be an exact image of the reality.

We find the following mean surgery durations:

- For one surgery per session:
  - $H_{1,1} = \{1\}$: $mean(H_{1,1}) = 227$.
  - $H_{2,1} = \{2\}$: $mean(H_{2,1}) = 534$.
- For two surgeries on the session:
  - $H_{1,2} = \{1,1\}$: $mean(H_{1,2}) = 571$.
  - $H_{2,2} = \{1,2\}$: $mean(H_{2,2}) = 830$.
  - $H_{3,2} = \{2,2\}$: $mean(H_{3,2}) = 1084$.

When we look at these mean surgery durations, we only have two possible ways of scheduling surgeries per session: One long surgery, or two medium long surgeries. Now, we only have one instead of two choices when scheduling one surgery per session, and one possibility instead of three when scheduling two surgeries per session. Thus, we have to re-normalize the probabilities, which are:

$$\mathbb{P}(H_{2,1}) = \frac{\mathbb{P}(V_2)}{\mathbb{P}(V_2) + \mathbb{P}(V_1)} = 1,$$
$$\mathbb{P}(H_{1,2}) = \frac{\mathbb{P}(V_1)^2}{\mathbb{P}(V_1)^2 + \mathbb{P}(V_1)\mathbb{P}(V_2) + \mathbb{P}(V_2)^2} = 1.$$

Now, we know the possible combinations of surgery duration groups and their probabilities. So, we can calculate the expected overtime, undertime, and utilization with the functions in Section 6.2.

Example 6.4. In the previous example, we have considered a surgical department with two groups of surgery durations. Here, we are going to look at the surgery duration data of HLK, and we choose $k = 3$: we have groups of short, medium long, and long surgery duration.

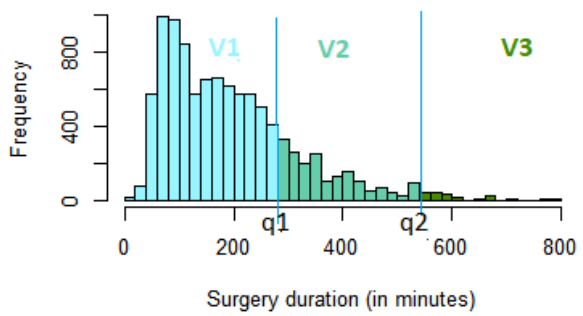


Figure 16: Example: Three groups of surgery durations

Here we have the probabilities for the groups:

$$\mathbb{P}(V_1) = \mathbb{P}(D \leq q_1) = 0.8028,$$
$$\mathbb{P}(V_2) = \mathbb{P}(q_1 < D \leq q_2) = 0.1840,$$
$$\mathbb{P}(V_3) = \mathbb{P}(D > q_2) = 0.0132,$$

See Section 3.4, HLK schedules either one, two, or three surgeries on session. Than, we would have $\binom{3+1-1}{3-1} + \binom{3+2-1}{3-1} + \binom{3+3-1}{3-1} = \binom{3}{2} + \binom{4}{2} + \binom{5}{2} = 19$ possible ways of scheduling surgeries. The mean durations of the groups and combinations of groups are:

- $H_{1,1} = \{1\},\ mean(H_{1,1}) = 277$.
  Scheduling one short surgery per session will be excluded, because it does not satisfy the requirement of a mean surgery duration of at least 300 minutes.
- $H_{2,1} = \{2\},\ mean(H_{2,1}) = 369$.
- $H_{3,1} = \{3\},\ mean(H_{3,1}) = 628$.
  The mean surgery duration of the long surgery duration group does not satisfy the requirements of begin at most 600 minutes. Therefore, we would have to exclude scheduling one long surgery on a session. However, we do have to include this surgery group at least in one of the possible combinations. Fortunately, the probability that we have a long surgery is very small. .
- $H_{1,2} = \{1,1\},\ mean(H_{1,2}) = 309$.
- $H_{2,2} = \{1,2\},\ mean(H_{2,2}) = 535$.
- $H_{3,2} = \{1,3\},\ mean(H_{3,2}) = 793$.
  We already knew, that scheduling one long surgery hardly fitted in one session. Therefore, we can definitely not schedule it with another surgery on a session.

- $H_{4,2} = \{2, 2\},\ mean(H_{4,2}) = 757.$
  The mean duration of one short surgery and one long surgery, and the mean duration of two medium surgeries per session, both including 20 minutes turnover time, are higher than 600 minutes. These combinations will be excluded.
  Automatically, all other the combinations of two surgeries are excluded, because the mean of $H_{3,2}$ and $H_{4,2}$ was already too long.
- $H_{1,3} = \{1, 1, 1\},\ mean(H_{1,3}) = 409.$
- $H_{2,3} = \{1, 1, 2\},\ mean(H_{2,3}) = 710.$
  Again, this mean duration is too long.

So, for HLK we have five ways of scheduling surgeries per session. We have to re-normalized the probabilities:

- For one surgery per session:
  - $\mathbb{P}(H_{1,2}) = \frac{\mathbb{P}(V_2)}{\mathbb{P}(V_2)+\mathbb{P}(V_3)} = 0.9330.$
  - $\mathbb{P}H_{1,3}) = \frac{\mathbb{P}(V_3)}{\mathbb{P}(V_2)+\mathbb{P}(V_3)} = 0.0670.$
- For two surgery per session:
  - $\mathbb{P}(H_{1,2}) = \frac{\mathbb{P}(V_1)^2}{\mathbb{P}(V_1)^2+\mathbb{P}(V_1)\mathbb{P}(V_2)} = 0.8135.$
  - $\mathbb{P}(H_{2,2}) = \frac{\mathbb{P}(V_1)\cdot\mathbb{P}(V_2)}{\mathbb{P}(D1)^2+\mathbb{P}(V_1)\mathbb{P}(V_2)} = 0.1865.$
- For three surgeries per session:
  - $\mathbb{P}(H_{1,3}) = 1.$

Next, we calculate the expected over- and undertime, and utilization for each combinations for different session lengths. Then, for one surgery per session, the expected overtime will be:

$$\mathbb{E}_1[o] = \mathbb{P}(V_2) \cdot \mathbb{E}_{V_2}[o] + \mathbb{P}(V_3) \cdot \mathbb{E}_{V_3}[o], \tag{21}$$

with $\mathbb{E}_{V_2}[o]$ and $\mathbb{E}_{V_3}[o]$, the expected overtime for, respectively, one medium long surgery and one long surgery per session. The same holds for undertime and utilization, and all combinations.

6.5 Determining the best session length

After determining the expected overtime, undertime, and corresponding utilization for each surgical department (considering the different number of surgeries per session), we want to conclude what the individual session length is for each surgical department. In Section 4.2 we have given the objectives, these will be our sub-scenarios. Recall:

-.1 Overtime and undertime have equal weight.

-.2 Overtime weights twice as much as undertime.

-.3 The utilization is minimal 85%, and the overtime is minimized.

Obviously, longer sessions induce less expected overtime, and more expected undertime. Therefore, we have decided that the best session length in the first two situations, is the length that satisfies the following:

$$\text{Sub-scenario 1}: \quad S_n = \operatorname{argmin}\{\mathbb{E}_n[u] + \mathbb{E}_n[o]\} \tag{22}$$

$$\text{Sub-scenario 2}: \quad S_n = \operatorname{argmin}\{\mathbb{E}_n[u] + 2 \cdot \mathbb{E}_n[o]\}, \tag{23}$$

with n the amount of surgeries per session considered.
We have determined Eqn. 22 and 23 for each department for the different amount of surgeries per session a department performs. For the third sub-scenario, we take the session length with minimal 85% utilization, and minimal overtime. For the third sub-scenario:

$$S_n = \operatorname{argmin}\{b_n \geq 85\%, \min\{\mathbb{E}_n[o]\}\}, \tag{24}$$

with b_n the utilization determined for n surgeries.
Due to the definition of utilization, a higher utilization means more overtime. Therefore, if we want to have a utilization higher than 85%, we have to be satisfied with a higher amount of overtime.

The first research question states: "*What is the optimal session length per surgical department, when they can only have one length?*". We are going to answer this question using the answers of the sub-scenarios. We could have been done this in two ways, 'independent' or 'dependent' of the amount of surgeries per session. In the first case, determining the best session length 'independent' of the amount of surgeries, the best session length concerning the first two sub-scenarios is the sum over all expectations for the different amounts of surgeries considered:

$$S = \operatorname{argmin}\left\{\sum_{n}^{i} \left(\mathbb{E}_n[u] + w \cdot \mathbb{E}_n[o]\right)\right\}. \tag{25}$$

Here, w ia the weight we assign to the overtime ($w = 1$ in the first scenario, and $w = 2$ in the second scenario). Note, n starts not necessarily at 1. Therefore, we have not specified what the minimal amount of surgeries is. Here, the determination of the best session length is not really independent of the amount of surgeries per session, but we do not take into account that probabilities of those amounts. Therefore, we have called this 'independent' of the amount of surgeries per session.

We rather do take the probability of the number of surgeries per session into account. These probabilities are given in Section 3.4.1. So, the best session length is determined in the following way:

$$S = \sum_{n}^{i} \mathbb{P}(n) S_n, \tag{26}$$

with $\mathbb{P}(n)$ the probability that there are n surgeries per session. Then, the utilization corresponding to the best session length S is:

$$b = \sum_{n}^{i} \mathbb{P}(n) b_n(S), \tag{27}$$

with $b_n(S)$ the utilization corresponding with the best session length S for n surgeries per session.

In the next chapter the results of both scenarios are given and discussed.

7 Results RQ1: The best session length

This chapter gives the results and conclusions on these results of the two scenarios, and the sub-scenarios, discussed in Section 6.4 and 6.5. With these results we will answer the first research question:

"*What is the optimal session length per surgical department, when they can only have one length?*".

In the following subsections we will completely elaborate the scenarios and the results for four departments, namely NCH, URO, KNO, and URK. We have decided to discuss these four departments, because we have started the research by thoroughly looking into the data of NCH and URO (see Section 5.1), KNO has a remarkable narrow and high top in its histogram of the surgery duration, and in Section 5.2.2 we have seen that the surgery durations of URK most likely follows a Gamma distributions. So, we will consider four different looking departments. We will give the best session length of each surgical department. Nevertheless, in Appendix C.5, the lists of expected overtime, undertime, and utilization for each departments can be found.

7.1 Scenario 1: Surgical duration groups by dividing the data

Recall from Section 6.4, the first way of dividing the surgery durations of the surgical departments in groups, we have made the groups using the R-function *cut(H$breaks,2)* or *cut(H$breaks,3)*. The complete written R-code that has been written for the calculations in this scenario can be found in Appendix D.5.

7.1.1 The combinations of surgery groups

In the histograms of the surgery durations, the surgery duration groups are given in different colors. One can find these histograms in Appendix C.4. Figure 17 gives these histograms of surgical departments NCH, URO, KNO, and URK.

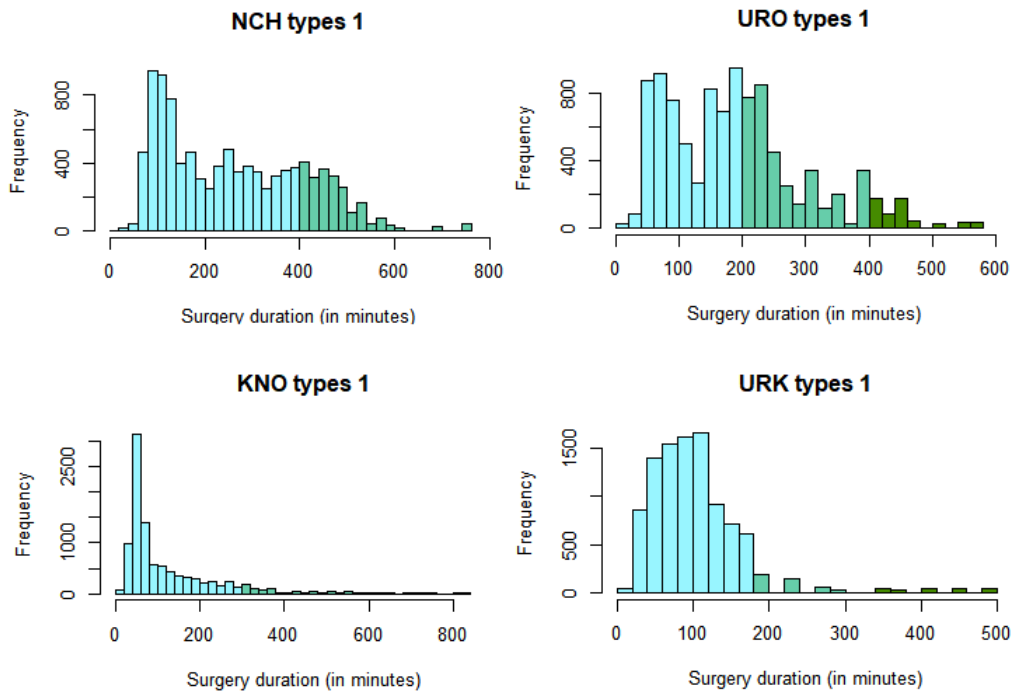


Figure 17: The groups (types 1) for NCH, URO, KNO, and URK

Now that we have composed the surgery duration groups of each department, we need to determine all allowed ways of scheduling surgeries per session. The requirements a surgery duration group or combination of surgery durations groups should meet to be seen at a possibility of scheduling surgeries, has been explained in Section 6.3. Unfortunately we have not managed to write a function in R that outputs all combinations allowed. Therefore, we have calculated the mean of 'every' combinations one at the time using R. Figure 18 gives the code to determine all ways of scheduling one or two surgeries of NCH per session. We have done this for each department, cf. Appendix D.5.

```
library(distr)
library(MASS)
set.seed(1234)
#NCHduur is the data of all surgery durations of NCH in 2015
gr<-(max(NCHduur)/2)
br<-gr-gr%%20+20 #the boundery of the two groups
#duration group 1
NCHduur1<-NCHduur[NCHduur<=br]
#duration group 2
NCHduur2<-NCHduur[NCHduur>br]

E<-EmpiricalDistribution(NCHduur) # the empirical probability distributions of NCH
r<-r(E)(10000) #10,000 drawings
y<-max(r)+20-max(r)%%20
H<-hist(r,breaks=c(seq(0,y,20)))
#the probabilities of the surgery duration
K<-data.frame(H$mids,sapply(H$counts,function(x) x/sum(H$counts)))
HH<-data.frame(H$mids,K)
D1<-colSums(HH[which(HH$H.mids<=br),])[2] #chance on group 1
D2<-colSums(HH[which(HH$H.mids>br),])[2] #chance on group 2

#the empirical probability distributions for 1 surgery on a session
E1<- EmpiricalDistribution(NCHduur1)
E2<-EmpiricalDistribution(NCHduur2)
r1<-r(E1)(10000) #10,000 drawings
r2<-r(E2)(10000)
y1<-max(r1)+20-max(r1)%%20 # to fix the 20 min intervals
y2<-max(r2)+20-max(r2)%%20
H1<-hist(r1,breaks=c(seq(0,y1,20)))
# mean of one medium long surgery
colSums(data.frame(H1$mids,H1$counts,H1$mids*H1$counts))[3]/10000
H2<-hist(r2,breaks=c(seq(br,y2,20)))
# mean of one long surgery
colSums(data.frame(H2$mids,H2$counts,H2$mids*H2$counts))[3]/10000
#the empirical probability distributions for 2 surgeries on a session
E11<-E1+E1
E12<-E1+E2
E22<-E2+E2
r11<-r(E11)(10000)
r11<-r11+20
r12<-r(E12)(10000)
r12<-r12+20
r22<-r(E22)(10000)
r22<-r22+20
y11<-max(r11)+20-max(r11)%%20
y12<-max(r12)+20-max(r12)%%20
y22<-max(r22)+20-max(r22)%%20
H11<-hist(r11,breaks=c(seq(0,y11,20)))
# mean of two medium long surgeries
colSums(data.frame(H11$mids,H11$counts,H11$mids*H11$counts))[3]/10000
H12<-hist(r12,breaks=c(seq(0,y12,20)))
# mean of one medium long and one long surgery
colSums(data.frame(H12$mids,H12$counts,H12$mids*H12$counts))[3]/10000
H22<-hist(r22,breaks=c(seq(0,y22,20)))
# mean of two long surgeries
colSums(data.frame(H22$mids,H22$counts,H22$mids*H22$counts))[3]/10000
```

Figure 18: Code to determine which combinations of surgery groups of NCH are allowed

Recall, in Section 3.4, we have given which amounts of surgeries per session are going to be taken into account at for each surgical department. In the table below we have given all allowed ways of scheduling one of more surgeries per session, determined with the R-code in Appendix D.5. Where:

1 = We are allowed to schedule
0 = Combination is not allowed to be scheduled
$H_{h,n}$ is a combination of surgery duration groups, and equal to a list of n numbers of $g = 1, .., k$.

Number of surgeries	1	1	1	2	2	2	2	3	3	3	3	3	3
$H_{r,n}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,1\}$	$\{1,2\}$	$\{1,3\}$	$\{2,2\}$	$\{1,1,1\}$	$\{1,1,2\}$	$\{1,1,3\}$	$\{1,2,2\}$	$\{1,2,3\}$	$\{2,2,2\}$
NCH	0	0	1	0	0	0	1	0	0	0	0	0	0
CCH	0	0	1	0	0	0	1	0	0	0	0	0	0
Gender	0	0	1	0	1	1	1	1	1	0	0	0	0
URO	0	0	1	0	1	1	1	1	1	0	0	0	0
HLK	0	1	1	1	1	0	0	1	0	0	0	0	0
LON	0	1	1	0	1	0	0	1	0	0	0	0	0
GYN	0	0	1	0	1	1	0	1	1	0	0	0	0
ORT	0	1	1	0	1	0	0	1	0	0	0	0	0
PCH	0	1	1	0	1	0	0	1	1	0	0	0	0
MKA	0	1	1	0	1	0	0	1	0	0	0	0	0
OOG	0	0	0	0	1	1	1	1	1	1	0	0	0
URK	0	0	0	0	1	1	1	1	1	0	0	0	0
KCH	0	0	0	0	0	0	0	1	1	1	1	1	1
KNO	0	1	1	0	1	1	0	1	1	0	0	0	0

Number	4	4	4	4	5	5	6	6	7
$H_{r,n}$	$\{1,1,1,1\}$	$\{1,1,1,2\}$	$\{1,1,1,3\}$	$\{1,1,2,2\}$	$\{1,1,1,1,1\}$	$\{1,1,1,1,2\}$	$\{1,1,1,1,1,1\}$	$\{1,1,1,1,1,2\}$	$\{1,1,1,1,1,1,1\}$
NCH	0	0	0	0	0	0	0	0	0
CCH	0	0	0	0	0	0	0	0	0
Gender	0	0	0	0	0	0	0	0	0
URO	0	0	0	0	0	0	0	0	0
HLK	0	0	0	0	0	0	0	0	0
LON	0	0	0	0	0	0	0	0	0
GYN	1	1	0	0	0	0	0	0	0
ORT	1	0	0	0	0	0	0	0	0
PCH	1	0	0	0	0	0	0	0	0
MKA	1	0	0	0	0	0	0	0	0
OOG	1	1	0	0	1	1	0	0	0
URK	1	1	0	0	1	0	0	0	0
KCH	1	1	1	1	1	1	1	1	1
KNO	1	1	0	0	1	0	0	0	0

Table 16: All ways of scheduling surgeries considered

7.1.2 Neurosurgery

For NCH, we know that the:
probability of 1 surgery per session: $\mathbb{P}(1) = 0.56$, and
probability of 2 surgeries per session: $\mathbb{P}(2) = 0.44$.

Further, the combinations of surgery duration groups that are allowed are:

- 1 long surgery, or
- 2 medium long surgeries.

In Table 17, we have given a list of all expected overtimes, undertimes, and utilization for different session lengths and one or two surgeries per session. From this data, we can calculated the current utilization (under scenario 1), when we would use the regular session length of 465 minutes as the only session length. We have rounded up to 470 minutes, to easily calculate the utilization from our results.

$$b = 0.56 \cdot 54.22\% + 0.4 \cdot 60.46\% \approx 54\%.$$

So for NCH, a session length of 470 minutes is not sufficient. We want to have a utilization of at least 85% and we also want to have a session length which minimizes the overtime.

In the next three paragraphs we give the calculations of the best session length and corresponding utilization for the three sub-scenario's given in Section 6.4.

S	$\mathbb{E}_1[u]$	$\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+\mathbb{E}_1[o]$	$\mathbb{E}_1[u(E2)]+2\mathbb{E}_1[o(E2)]$	b_1	$\mathbb{E}_2[u]$	$\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+2\mathbb{E}_2[o]$	b_2
290	0.00	173.38	173.38	346.76	159.79%	12.82	128.71	141.53	270.23	127.27%
310	0.00	153.38	153.38	306.76	149.48%	17.89	113.78	131.67	245.44	117.12%
330	0.00	133.38	133.38	266.76	140.42%	23.71	99.60	123.31	222.91	108.26%
350	0.00	113.38	113.38	226.76	132.39%	30.42	86.30	116.72	203.02	99.84%
370	0.00	93.38	93.38	186.76	125.24%	38.01	73.89	111.90	185.80	92.27%
390	0.00	73.38	73.38	146.76	118.81%	46.72	62.61	109.33	171.93	84.47%
410	2.80	56.18	58.98	115.16	100.38%	56.57	52.45	109.02	161.47	77.37%
430	8.72	42.10	50.83	92.93	82.21%	67.49	43.37	110.86	154.23	71.18%
450	16.89	30.27	47.16	77.43	69.64%	79.48	35.36	114.84	150.20	65.56%
470	28.16	21.54	49.70	71.24	54.22%	92.53	28.42	120.95	149.36	60.46%
490	41.85	15.22	57.07	72.29	43.23%	106.81	22.69	129.50	152.19	55.06%
510	57.66	11.04	68.69	79.73	34.41%	122.09	17.98	140.07	158.04	51.03%
530	74.48	7.86	82.35	90.21	31.41%	138.23	14.11	152.34	166.45	48.06%
550	92.70	6.08	98.78	104.86	26.88%	155.02	10.91	165.93	176.83	46.20%
570	111.30	4.68	115.97	120.65	27.36%	172.39	8.27	180.66	188.93	44.86%
590	130.46	3.84	134.29	138.13	26.96%	190.22	6.10	196.32	202.42	44.13%
610	149.77	3.15	152.92	156.07	28.51%	208.45	4.33	212.78	217.11	43.72%

Table 17: expectation in min, utilization percentage NCH - type 1

7.1.2.1 Scenario 1.1: Under- and overtime weigh equally

From the results of our simulations, given in Table 17 we get using Eqn. 22:

- For one surgery per session: $S_1 = 450$ minutes.
- For two surgeries per session: $S_2 = 410$ minutes.

Therefore, when we give undertime and overtime the same weigth, the best ssion length is:

$$S = 0.56 \cdot 450 + 0.44 \cdot 410 \approx 432 \text{ minutes}.$$

For the session length $S = 432$, we have the following utilization $b \approx 77\%$.

7.1.2.2 Scenario 1.2: Overtime weighs twice as much as undertime

When we give overtime a weight twice as much as undertime, we get the following session lengths using Eqn. 23:

- For one surgery per session: $S_1 = 470$ minutes.
- For two surgeries per session: $S_2 = 470$ minutes.

We see that in this case, the best session length is $S = 470$ for both scheduling one or two surgeries per session. This gives a utilization $b \approx 54\%$.

7.1.2.3 Scenario 1.3: Minimal utilization of 85% and minimal overtime

In the last sub-scenario we want to have the session length where we have a utilization of at least 85%. Due to our definition of utilization (see Eqn. 9), the utilization is higher for a larger amount of overtime. However, we also want to minimize the overtime, thus the overtime has to be as low as possible with a utilization of at least 85%. Therefore, we get the following best session length of Table 17:

- For one surgery per session: $S_1 \approx 430$ minutes.
- For two surgeries per session: $S_2 \approx 390$ minutes.

In this case, it follows that the best session length is

$$S = 0.56 \cdot 430 + 0.44 \cdot 390 \approx 412 \text{ minutes.}$$

Then, we have a utilization $b \approx 90\%$.

7.1.2.4 Scenario 1 - NCH: Conclusions

As discussed before, it is far more logical to give the overtime a higher weight than undertime, because overtime comes with more cost such as higher wages. However, assigning NCH a session length based on scenario 1.2, leaves us with a very low utilization. The best session length under scenario 1.2 was the current regular session length. So, we can immediately conclude that however we reckon the extra cost of overtime, a session length $S = 470$ is not satisfactory. Moreover, scenario 1.3 has given a utilization $b = 90\%$, so we would have to conclude that the best session length for NCH, in this case, 412 minutes is. So, almost an hour shorter than the regular session length of 465 minutes.

Interesting is, for a couple of years now, NCH has insisted to get longer session lengths. Assumable, the departments wants longer session so they can schedule three instead of one or two surgeries per session. We have not taken into account the case of three surgeries per session for NCH, because it hardly occured in 2015. However, the best session length of 412 minutes, shows that NCH will suffer undertime when we have a session length of 465 minutes. Therefore, a session length longer than 465 minutes makes it possible to schedule three surgeries per session, most likely.

7.1.3 Urology

In Section 3.4, we have given that URO either schedules one, two, or three surgeries. The probabilities on these amounts are:
the probability of 1 surgery: $\mathbb{P}(1) = 0.23$,
the probability of 2 surgeries: $\mathbb{P}(2) = 0.61$, and
the probability of 3 surgeries: $\mathbb{P}(3) = 0.17$.

We have determined that the following surgery duration groups combinations are allowed to be scheduled:

- 1 long surgery,
- 1 short surgery + 1 medium surgery,
- 1 short surgery + 1 long surgery,
- 2 medium long surgeries,
- 3 short surgeries, and
- 2 short surgeries + 1 medium long surgery.

The results of our simulations are given in Table 18.
We have rounded up the regular session length to $S = 470$ minutes, so that is applies to our results. For $S = 470$, we have $b \approx 63\%$.

7.1.3.1 Scenario 1.1: Under- and overtime weigh equally

First, we have looked at the best session length when overtime and undertime have equal weight:

- For one surgery per session: $S = 450$ minutes.
- For two surgeries per session: $S = 470$ minutes.
- For three surgeries per session: $S = 450$ minutes.

Thus, in this case the best session length for URO would be:

$$S = 0.23 \cdot 450 + 0.61 \cdot 470 + 0.17 \cdot 450 \approx 467 \text{ minutes.}$$

From Table 18, it follows that for $S = 467$ minutes. The utilization is approximately $b \approx 63\%$. We can conclude that the regular session length of 465 minutes is suited, when we weigh overtime and undertime equally.
However, we have mentioned that giving overtime a higher weight than undertime is more in line with the reality. The next paragraph gives the results for that situation.

7.1.3.2 Scenario 1.2: Overtime weighs twice as much as undertime

We have:

- For one surgery per session: $S = 450$ minutes.
- For two surgeries per session: $= 530$ minutes.
- For three surgeries per session: $S = 490$ minutes.

Then follows, that the best session length is:

$$S = 0.23 \cdot 450 + 0.61 \cdot 530 + 0.17 \cdot 490 \approx 510 \text{ minutes.}$$

So, we get a utilization of $b \approx 52\%$. This utilization is much lower than in the case of equal weighting, but that will automatically follows from the definitions 9 we use.

7.1.3.3 Scenario 1.3: Minimal utilization of 85% and minimal overtime

The regular session length, and both situations discussed above, still do not give URO a utilization of at least 85%. For at least 85% utilization, we have gotten for the different amount of surgeries we have considered, the following session length:

- For one surgery per session: $S \approx 420$ minutes
- For two surgeries per session: $S \approx 430$ minutes
- For three surgeries per session: $S \approx 410$ minutes

This encounters the session length:

$$S \approx 0.23 \cdot 420 + 0.61 \cdot 430 + 0.17 \cdot 410 \approx 429 \text{ minutes},$$

with utilization $b \approx 86\%$.

7.1.3.4 Scenario 1 - URO: Conclusions

As well as for NCH, we see that when we want to have a utilization of at least 85% we can not minimize the expected overtime. Also, URO also needs a shorter session length then 465 minutes to deliver an expected utilization of at least 85%. However, the difference with the current regular session length is a only a bit more than 30 minutes. So, one could argue whether changing the session length is worth the effort.

S	$\mathbb{E}_1[u]$	$\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+$ $\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+$ $2\mathbb{E}_1[o]$	b_1	$\mathbb{E}_2[u]$	$\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+$ $\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+$ $2\mathbb{E}_2[o]$	b_2	$\mathbb{E}_3[u]$	$\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+$ $\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+$ $2\mathbb{E}_3[o]$	b_3
290	0.00	162.00	162.00	324.01	155.86%	0.24	185.75	186.00	371.75	163.07%	3.47	163.20	166.66	329.86	150.22%
310	0.00	142.00	142.00	284.01	145.81%	1.08	166.59	167.67	334.25	149.91%	5.46	145.19	150.66	295.85	138.62%
330	0.00	122.00	122.00	244.01	136.97%	2.87	148.38	151.24	299.62	136.89%	8.18	127.90	136.08	263.98	127.68%
350	0.00	102.00	102.00	204.01	129.14%	5.58	131.09	136.66	267.75	125.50%	11.77	111.50	123.28	234.78	117.23%
370	0.00	82.00	82.00	164.01	122.16%	9.10	114.61	123.72	238.33	115.80%	16.51	96.24	112.75	209.00	106.78%
390	0.00	62.00	62.00	124.01	115.90%	13.47	98.98	112.45	211.43	107.01%	22.41	82.14	104.54	186.68	97.34%
410	0.00	42.00	42.00	84.01	110.24%	18.82	84.33	103.16	187.49	98.38%	29.51	69.24	98.75	167.99	88.56%
430	5.86	27.87	33.73	61.60	78.52%	25.36	70.87	96.23	167.09	89.71%	37.87	57.60	95.48	153.08	80.39%
450	15.12	17.12	32.24	49.36	60.90%	33.25	58.76	92.01	150.76	80.99%	47.50	47.23	94.73	141.96	72.92%
470	30.55	12.56	43.11	55.66	31.99%	42.58	48.09	90.67	138.76	72.64%	58.48	38.21	96.69	134.89	65.68%
490	47.09	9.10	56.19	65.29	28.76%	53.38	38.88	92.26	131.14	64.85%	70.75	30.47	101.22	131.69	59.32%
510	63.64	5.64	69.28	74.92	30.87%	65.56	31.06	96.62	127.68	58.05%	84.24	23.97	108.21	132.18	53.74%
530	81.37	3.37	84.74	88.11	27.33%	78.86	24.37	103.24	127.61	52.93%	98.79	18.51	117.30	135.81	49.41%
550	99.10	1.10	100.20	101.31	29.56%	93.28	18.79	112.06	130.85	48.31%	114.32	14.05	128.38	142.43	45.65%
570	118.00	0.00	118.00	118.00	26.22%	108.56	14.07	122.64	136.71	45.08%	130.73	10.46	141.19	151.65	42.74%
590	138.00	0.00	138.00	138.00	23.39%	124.70	10.21	134.91	145.12	42.18%	147.92	7.64	155.56	163.21	40.44%
610	158.00	0.00	158.00	158.00	25.90%	141.62	7.13	148.75	155.88	39.79%	165.71	5.44	171.15	176.58	39.09

Table 18: expectation in min, utilization percentage URO - type 1

7.1.4 Ear, nose, and throat surgery (KNO)

For KNO we have simulated the expected overtime, undertime, and utilization for one, two, three, four, or five surgeries on session. The probabilities of these amounts are:
the probability of 1 surgery: $\mathbb{P}(1) = 0.18$,
the probability of 2 surgeries: $\mathbb{P}(2) = 0.29$,
the probability of 3 surgeries: $\mathbb{P}(3) = 0.19$,
the probability of 4 surgeries: $\mathbb{P}(4) = 0.20$, and
the probability of 5 surgeries: $\mathbb{P}(5) = 0.14$.

The combinations of surgery duration groups allowed:

- 1 medium long surgery,
- 1 long surgery,
- 1 short surgery + 1 medium long surgery,
- 1 short surgery + 1 long surgery,
- 3 short surgeries,
- 2 short surgeries + 1 medium long surgery,
- 4 short surgeries,
- 3 short surgeries + 1 medium long surgery,
- 5 short surgeries.

Table 19 gives a list of expected overtimes, undertimes, and utilizations for one or two surgeries per session. For three, four, and five surgeries per sessions, the results can be found in Table 20.
In this scenario, the current regular session length will give a utilization of $b \approx 66\%$.

Again, we have elaborated the three sub-scenario's for KNO.
Recall, here we look at session lengths $300 \leq S \leq 720$.

7.1.4.1 Scenario 1.1: Under- and overtime weigh equally

Using Eqn. 22, we get the following session lengths for the different amount of surgeries considered separately:

- For one surgery per session: $S = 370$ minutes.
- For two surgeries per session: $S = 450$ minutes.
- For three surgeries per session: $S = 310$ minutes.
- For four surgeries per session: $S = 430$ minutes.
- For five surgeries per session: $S = 530$ minutes.

This results in the following session length:

$$S = 0.18 \cdot 370 + 0.29 \cdot 450 + 0.19 \cdot 310 + 0.20 \cdot 430 + 0.14 \cdot 530 \approx 416 \text{ minutes},$$

with an expected utilization of $b \approx 77\%$.
We have in this case a utilization closer to 85%. We will to look into the other two sub-scenario's to find an even beter utilization.

7.1.4.2 Scenario 1.2: Overtime weighs twice as much as undertime

Here we use Eqn. 23, and we find the following session length for the different amount of surgeries.

- For one surgery per session: $S = 450$ minutes.
- For two surgeries per session: $S = 510$ minutes.
- For three surgeries per session: $S = 390$ minutes.
- For four surgeries per session: $S = 490$ minutes.
- For five surgeries per session: $S = 590$ minutes.

Yielding:

$$S = 0.18 \cdot 450 + 0.29 \cdot 510 + 0.19 \cdot 390 + 0.20 \cdot 490 + 0.14 \cdot 590 \approx 584 \text{ minutes},$$

with $b \approx 47\%$.

Here, we have given the overtime a higher weight than undertime, meaning we rather have some undertime than overtime. Therefore, we need a longer session length to catch up with the overtime. Due to the definitions of utilization (Eqn. 9), automatically we get a lower utilization when we take into account more overtime.

7.1.4.3 Scenario 1.3: Minimal utilization of 85% and minimal overtime

In the last sub-scenario, we want to minimize the overtime and have a utilization of at least 85%. This holds for the following session lengths considering the different amounts of surgeries per session:

- For one surgery per session: $S \approx 350$ minutes.
- For two surgeries per session: $S \approx 430$ minutes.
- For three surgeries per session: $S \approx 310$ minutes.
- For four surgeries per session: $S \approx 410$ minutes.
- For five surgeries per session: $S \approx 490$ minutes.

The best session length in this case would be:

$$S \approx 0.18 \cdot 350 + 0.29 \cdot 440 + 0.19 \cdot 310 + 0.20 \cdot 410 + 0.14 \cdot 490 \approx 400 \text{ minutes}.$$

A session length $S = 400$ gives an expected utilization of $b \approx 86\%$.

In the last paragraph of this section, we have given some general conclusions of the best session length for KNO.

7.1.4.4 Scenario 1 - KNO: Conclusions

We can conclude that for surgical department KNO it is very hard to minimize the overtime, and also make sure we have a utilization of at least 85%. When we only minimize the overtime while the utilization is at least 85%, the best session length for KNO would be 400 minutes. However, we have a large difference between the session length in scenario 1.2 and scenaro 1.2, therefore we have to conclude that for $S = 400$ we might have much overtime.
Further, in the session distribution of 2017, KNO gets an extra long session of 705 minutes every week. We can conclude that it might be insufficient to assign KNO only one session length, and rather assign multiple. More on this in Section 9.

S	$\mathbb{E}_1[u]$	$\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+2\mathbb{E}_1[o]$	b_1	$\mathbb{E}_2[u]$	$\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+2\mathbb{E}_2[o]$	b_2
290	0.00	128.01	128.01	256.02	144.14%	0.00	472.36	174.98	349.96	160.34%
310	2.48	110.50	112.98	223.48	124.02%	0.01	432.37	154.99	309.97	149.97%
330	8.15	96.16	104.30	200.46	103.30%	0.17	392.53	135.31	270.46	140.19%
350	16.01	84.02	100.02	184.04	89.27%	1.28	353.64	117.55	233.81	128.00%
370	25.70	73.71	99.40	173.11	78.42%	4.23	316.59	103.43	202.63	113.24%
390	36.98	64.99	101.98	166.97	69.71%	9.16	281.52	93.29	177.43	99.26%
410	48.83	56.84	105.67	162.51	66.54%	16.01	248.37	87.00	157.99	86.95%
430	61.18	49.19	110.36	159.55	63.94%	24.56	216.92	84.10	143.64	76.81%
450	74.19	42.20	116.39	158.59	60.81%	34.48	186.84	83.93	133.38	69.08%
470	87.54	35.55	123.10	158.65	59.41%	45.53	157.92	86.03	126.54	63.05%
490	102.02	30.03	132.06	162.09	54.55%	57.64	130.53	90.25	122.86	57.88%
510	116.85	24.86	141.70	166.56	53.67%	70.59	105.50	96.15	121.72	54.09%
530	132.78	20.79	153.57	174.37	49.30%	84.48	83.55	103.94	123.40	50.15%
550	149.25	17.26	166.51	183.77	47.93%	99.34	64.69	113.66	127.98	46.37%
570	166.27	14.28	180.55	194.83	46.58%	115.29	48.57	125.56	135.83	42.28%
590	183.66	11.67	195.32	206.99	46.18%	132.32	34.92	139.62	146.92	38.50%
610	201.41	9.42	210.84	220.26	45.78%	150.29	23.89	155.55	160.81	35.68%
630	219.53	7.54	227.08	234.62	45.44%	168.84	15.90	172.67	176.49	34.62%
650	237.83	5.84	243.68	249.52	45.99%	187.76	10.44	190.50	193.24	34.73%
670	256.62	4.63	261.25	265.88	45.06%	206.91	7.03	208.80	210.69	35.41%
690	275.40	3.41	278.82	282.23	46.48%	226.27	4.63	227.52	228.77	36.17%
710	294.36	2.37	296.74	299.11	47.00%	245.81	2.83	246.59	247.37	37.05%

Table 19: expectation in min, utilization percentage KNO - type 1 (1/2)

S	$\mathbb{E}_3[u]$	$\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+2\mathbb{E}_3[o]$	b_3	$\mathbb{E}_4[u]$	$\mathbb{E}_4[o]$	$\mathbb{E}_4[u]+\mathbb{E}_4[o]$	$\mathbb{E}_4[u]+2\mathbb{E}_4[o]$	b_4	$\mathbb{E}_5[u]$	$\mathbb{E}_5[o]$	$\mathbb{E}_5[u]+\mathbb{E}_5[o]$	$\mathbb{E}_5[u]+2\mathbb{E}_5[o]$	b_5
290	29.02	79.50	108.52	188.02	97.22%	3.99	165.85	169.84	335.70	147.75%	0.06	250.12	250.17	500.29	186.02%
310	38.29	68.77	107.06	175.83	88.18%	7.12	148.98	156.10	305.08	134.71%	0.25	230.31	230.56	460.86	173.41%
330	48.67	59.15	107.82	166.97	80.77%	11.31	133.18	144.49	277.67	122.81%	0.80	210.86	211.65	422.51	161.40%
350	60.08	50.57	110.65	161.22	74.53%	16.59	118.45	135.04	253.50	112.20%	1.92	191.98	193.90	385.87	149.79%
370	72.49	42.97	115.46	158.43	69.18%	22.96	104.82	127.78	232.60	102.69%	3.77	173.83	177.60	351.43	138.74%
390	85.81	36.29	122.10	158.39	64.71%	30.45	92.31	122.75	215.06	94.04%	6.43	156.49	162.92	319.42	128.47%
410	100.09	30.57	130.66	161.22	60.48%	38.92	80.78	119.70	200.47	86.84%	9.98	140.04	150.03	290.07	118.83%
430	115.17	25.65	140.82	166.48	57.33%	48.45	70.32	118.77	189.09	79.93%	14.41	124.47	138.88	263.36	110.16%
450	130.98	21.47	152.45	173.92	54.82%	59.03	60.89	119.92	180.80	73.79%	19.81	109.87	129.67	239.54	101.85%
470	147.40	17.88	165.28	183.17	53.08%	70.57	52.43	122.99	175.42	68.48%	26.19	96.26	122.46	218.71	94.09%
490	164.37	14.85	179.22	194.08	51.72%	83.11	44.97	128.07	173.04	63.43%	33.57	83.63	117.20	200.82	87.07%
510	181.79	12.27	194.07	206.34	50.94%	96.49	38.36	134.85	173.20	59.50%	42.12	72.18	114.29	186.47	79.67%
530	199.60	10.08	209.68	219.77	50.52%	110.77	32.63	143.41	176.04	55.66%	51.77	61.83	113.60	175.44	73.15%
550	217.75	8.23	225.97	234.20	50.36%	125.76	27.62	153.37	180.99	52.97%	62.54	52.60	115.14	167.75	67.09%
570	236.19	6.67	242.87	249.54	50.38%	141.40	23.27	164.67	187.94	50.64%	74.37	44.43	118.80	163.24	61.69%
590	254.89	5.37	260.26	265.63	50.62%	157.67	19.53	177.20	196.73	48.71%	87.22	37.28	124.49	161.77	56.88%
610	273.80	4.28	278.08	282.36	51.04%	174.46	16.33	190.79	207.12	47.30%	100.89	30.95	131.85	162.80	53.22%
630	292.90	3.38	296.28	299.66	51.54%	191.76	13.62	205.38	219.01	46.12%	115.45	25.51	140.95	166.46	49.61%
650	312.19	2.68	314.87	317.55	51.96%	209.42	11.28	220.70	231.97	45.67%	130.80	20.86	151.66	172.53	46.55%
670	331.61	2.09	333.70	335.79	52.73%	227.44	9.30	236.75	246.05	45.21%	146.86	16.92	163.78	180.70	44.15%
690	351.14	1.62	352.76	354.38	53.47%	245.77	7.64	253.41	261.05	45.07%	163.58	13.64	177.22	190.85	42.09%
710	370.77	1.26	372.03	373.29	54.23%	264.40	6.26	270.66	276.92	45.00%	180.79	10.85	191.64	202.50	40.92%

Table 20: expectation in min, utilization percentage KNO - type 1 (2/2)

7.1.5 Pediatric Urology (URK)

Surgical department URK schedules either two, three, four, or five surgeries per session, with the following probabilities:
Probability of 2 surgeries: $\mathbb{P}(2) = 0.14$.
Probability of 3 surgeries: $\mathbb{P}(3) = 0.43$.
Probability of 4 surgeries: $\mathbb{P}(4) = 0.25$.
Probability of 5 surgeries: $\mathbb{P}(5) = 0.18$.

The combinations of surgery duration groups allowed:

- 1 short surgery + 1 medium long surgery,
- 1 short surgery + 1 long surgery,
- 2 medium long surgeries,
- 3 short surgeries,
- 2 short surgeries + 1 medium long surgeries,
- 4 short surgeries,
- 3 short surgeries + 1 medium long surgery,
- 5 short surgeries.

With the current regular session length, URK has a utilization of 76% in this scenario.
Now, we are going to analyze the three sub-scenario's.

7.1.5.1 Scenario 1.1: Under- and overtime weigh equally

From the result in Table 21, we have that for the different amount of surgeries $\mathbb{E}[u] + \mathbb{E}[o]$ minimal is:

- For two surgeries per session: $S = 370$ minutes.
- For three surgeries per session: $S = 390$ minutes.
- For four surgeries per session: $S = 430$ minutes.
- For five surgeries per session: $S = 550$ minutes.

Then, the best session length is:

$$S = 0.14 \cdot 370 + 0.43 \cdot 390 + 0.25 \cdot 430 + 0.18 \cdot 550 \approx 426 \text{ minutes.}$$

For $S = 426$, the utilization is approximately 97%.

We want a utilization of at least 85%, so with a session length of 426 minutes URK would use their allocated OR-time excellently. However, in this case we have given overtime and undertime the same weight. As discussed before, it is much more realistic to give overtime an higher weight, due to the higher costs that come with overtime. Therefore, we look at the next case as well.

7.1.5.2 Scenario 1.2: Overtime weighs twice as much as undertime

In the previous case we had a very high utilization. We also want to look at the more realistic case of overtime having a weight twice as much as undertime. Now, we have the following session lengths using Eqn. 23:

- For two surgeries per session: $S = 410$ minutes.
- For three surgeries per session: $S = 430$ minutes.
- For four surgeries per session: $S = 470$ minutes.
- For five surgeries per session: $S = 590$ minutes.

The best session length here is:

$$S = 0.14 \cdot 410 + 0.43 \cdot 430 + 0.25 \cdot 470 + 0.18 \cdot 590 \approx 466 \text{ minutes},$$

The session length $S = 466$ is the current regular session length, and gives a utilization of 76%. We see that the more realistic case gives a lower utilization, this is due to the definition of utilization: More overtime implies a higher utilization. Therefore, minimizing the overtime will induce a lower utilization. Note, a utilization of 76% is still quite good, but we rather want a utilization of at least 85%.

7.1.5.3 Scenario 1.3: Minimal utilization of 85% and minimal overtime

We already have a good utilization with the first two sub-scenario's. For $S = 426$ we have a utilization of 97%, but here the overtime will not be as low as possible because a higher utilization is due to a higher amount of overtime. So, we look at the case where the utilization is 85%. This gives:

- For two surgeries per session:$S \approx 350$ minutes.
- For three surgeries per session: $S \approx 470$ minutes.
- For four surgeries per session: $S \approx 410$ minutes.
- For five surgeries per session: $S \approx 510$ minutes.

Which give the following session length:

$$S \approx 0.14 \cdot 350 + 0.43 \cdot 470 + 0.25 \cdot 410 + 0.18 \cdot 510 \approx 445 \text{ minutes},$$

and $b \approx 87\%$.

7.1.5.4 Scenario 1 - URK: Conclusions

The department URK has in each of the scenarios a fine to great utilization. The session length ranges from 426 to 466 minutes, so we can conclude that the current session length of 465 minutes is satisfactory, but could be shorter. The utilization is the highest for $S = 426$ minutes, but the overtimes is lowest for $S = 466$ minutes. The head of the OR departments should start the discussion whether a low overtime of a high utilization is more important.

S	$\mathbb{E}_2[u]$	$\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+2\mathbb{E}_2[o]$	b_2	$\mathbb{E}_3[u]$	$\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+2\mathbb{E}_3[o]$	b_3
290	3.75	102.99	106.74	209.73	124.89%	14.87	198.23	213.11	411.34	234.77%
310	7.97	87.20	95.17	182.37	109.64%	22.64	167.94	190.58	358.51	212.97%
330	13.99	73.23	87.22	160.45	96.30%	32.67	139.90	172.57	312.47	192.49%
350	22.13	61.36	83.49	144.86	83.17%	45.34	114.49	159.83	274.32	172.69%
370	32.04	51.27	83.31	134.58	72.98%	60.88	91.96	152.84	244.80	153.96%
390	43.48	42.71	86.19	128.90	64.89%	79.17	72.18	151.35	223.53	137.72%
410	56.05	35.29	91.34	126.63	59.39%	100.54	55.48	156.02	211.50	121.55%
430	69.63	28.86	98.49	127.35	55.04%	124.71	41.58	166.29	207.87	108.18%
450	84.00	23.23	107.23	130.47	51.97%	151.58	30.38	181.96	212.33	96.45%
470	99.02	18.25	117.27	135.53	49.85%	180.87	21.59	202.47	224.06	86.98%
490	114.77	14.01	128.78	142.78	47.52%	212.24	14.89	227.13	242.03	79.87%
510	131.13	10.36	141.49	151.85	45.96%	245.45	10.03	255.48	265.51	74.41%
530	148.03	7.27	155.30	162.56	44.79%	280.05	6.56	286.61	293.17	71.42%
550	165.56	4.79	170.35	175.14	43.34%	315.65	4.09	319.74	323.82	70.51%
570	183.75	2.98	186.72	189.70	41.82%	352.09	2.45	354.54	357.00	70.36%
590	202.47	1.70	204.16	205.86	41.00%	389.12	1.41	390.53	391.93	71.42%
610	221.66	0.89	222.55	223.44	40.52%	426.56	0.77	427.33	428.10	73.22%

S	$\mathbb{E}_4[u]$	$\mathbb{E}_4[o]$	$\mathbb{E}_4[u]+\mathbb{E}_4[o]$	$\mathbb{E}_4[u]+2\mathbb{E}_4[o]$	b_4	$\mathbb{E}_5[u]$	$\mathbb{E}_5[o]$	$\mathbb{E}_5[u]+\mathbb{E}_5[o]$	$\mathbb{E}_5[u]+2\mathbb{E}_5[o]$	b_5
290	0.75	149.14	149.89	299.03	149.42%	0.01	257.92	257.93	515.86	188.90%
310	1.60	129.99	131.59	261.58	138.19%	0.04	237.96	238.00	475.95	176.61%
330	3.13	111.52	114.65	226.17	127.10%	0.11	218.02	218.13	436.16	165.76%
350	5.52	93.91	99.43	193.35	116.45%	0.25	198.16	198.41	396.58	155.99%
370	9.16	77.55	86.71	164.26	105.25%	0.54	178.45	178.99	357.44	146.94%
390	14.22	62.61	76.82	139.43	94.41%	1.07	158.98	160.05	319.03	138.39%
410	20.94	49.34	70.28	119.62	83.50%	1.97	139.88	141.85	281.73	130.10%
430	29.50	37.89	67.39	105.28	72.89%	3.47	121.38	124.85	246.24	121.53%
450	39.93	28.32	68.25	96.58	63.01%	5.81	103.73	109.54	213.27	112.62%
470	52.26	20.66	72.92	93.57	53.86%	9.20	87.11	96.31	183.42	103.58%
490	66.27	14.67	80.94	95.61	46.46%	13.76	71.67	85.43	157.10	94.62%
510	81.83	10.22	92.05	102.28	40.27%	19.80	57.71	77.51	135.23	84.99%
530	98.58	6.98	105.56	112.54	36.15%	27.46	45.37	72.83	118.20	75.45%
550	116.24	4.63	120.87	125.50	33.71%	36.84	34.76	71.60	106.36	66.09%
570	134.64	3.04	137.68	140.72	32.12%	47.93	25.84	73.77	99.61	57.52%
590	153.53	1.92	155.46	157.38	31.91%	60.82	18.73	79.55	98.29	49.02%
610	172.81	1.21	174.02	175.22	32.13%	75.29	13.21	88.50	101.70	42.15%

Table 21: expectation in min. utilization percentage URK - type 1

7.1.6 Scenario 1: All surgical departments

In the previous section we have elaborated the three sub-scenario's for NCH, URO, HLK, and URK. All these departments had a higher utilization for a session length for a length lower than 465 minutes, which can be explain with the definition of utilization (Eqn. 9). For NCH, and URO the utilization was highest in scenario 1.3, where we wanted a utilization of at least 85% and the overtime as low as possible. The other two departments, KNO and URK, have the highest utilization in the case where we minimized the sum of the expected overtime and undertimes, with the same weight. We have also concluded that one session is supposedly not sufficient for KNO, because the departments has already some extra long sessions. Lastly, the current regular session length of 465 minutes is quite good for URK.

We have executed scenario 1 for all departments. Recall, in this scenario we have made the groups by dividing the surgery durations data by two of three. Then, when we only schedule with the current regular session length of 465 minutes, we have the following utilization for the surgical departments, see Table 22. Remark, we have rounded up to 470 minutes, to easily calculate the utilization from our results.

Department	Utilization
NCH	54 %
CCH	100 %
Gender	44 %
URO	63 %
HLK	77 %
LON	59 %
GYN	44 %
ORT	82 %
PCH	97 %
MKA	83 %
OOG	48 %
URK	76 %
KCH	70 %
KNO	66 %

Table 22: Scenario 1: Utilization for $S = 470$

From Table 22, we can conclude that for the surgical departments NCH, Gender, LON, GYN, and OOG, a session length of 470 minutes is not sufficient. There will be little overtime, but the utilization is very low. In the session distribution of 2017, NCH has been assigned an extended session every two weeks, these other departments have not. On the other hand, PCH and CCH have a very high utilization. This means they use their allocated OR-time sufficient, but is could also mean they have a large amount of overtime. The latter implies that these two departments need a longer session length. All other departments have a relatively high utilization, but lower than 85%. Therefore, a slight improvement of their session lengths is desirable.

In Appendix C.5, all list of expected undertime, overtime, and utilization for different session lengths and different amounts of surgeries per session are given. Below, we have given the best session length (in minutes) for each scenario with the corresponding utilization. We have used Eqns. 26, and 24, to determine the best session length for each sub-scenario. The next table gives these session lengths for each sub-scenario with the corresponding utilization (Eqn. 27).

Department	1.1S	1.1b	1.2S	1.2b	1.3S	1.3b
NCH	432	77%	470	54%	412	90%
CCH	555	66%	587	55%	501	82%
Gender	410	70%	445	50%	376	80%
URO	467	63%	510	52%	429	86%
HLK	370	81%	325	100%	358	83%
LON	413	79%	455	64%	389	86%
GYN	412	60%	461	46%	363	86%
ORT	491	78%	514	70%	459	73%
PCH	507	78%	559	64%	480	85%
MKA	479	83%	524	62%	460	87%
OOG	379	74%	411	67%	364	84%
URK	426	97%	466	76%	445	87%
KCH	481	67%	516	48%	449	80%
KNO	416	75%	584	47%	400	86%

Table 23: Scenario 1: Best session length for each sub-scenario.

We can conclude from Table 23 that for most surgical departments the utilization is the highest in scenario 1.3. This is not astonishing, because in scenario 1.3 we wanted a utilization of minimal 85%. Also, in scenario's 1.1 and 1.1 we have intended to minimize the overtime, and recall the utilization is lower for less overtime.
We can conclude for CCH and PCH, that a higher session length than 465 minutes should be assigned. Here, we have to note that in the session distribution of 2017, PCH has already been assigned an extra long session of 705 minutes every four weeks. Also, CCH has a longer session of 585 minutes almost every day. We see that that $S = 585$ corresponds with the best session length in case of scenario 1.2 with a utilization of 55%. This utilization is much lower then we prefer. Lastly, we see that departments NCH, Gender, LON, GYN, and OOG should get shorter session lengths in this case to make sure the balance between a high utilization and low overtimes is well enough.

Lastly, we have to remark that the assumption that a surgery group or a combination of surgery groups is only allowed to be scheduled when the mean surgery duration is more than 300 or less than 600, is not applicable for every surgical department. However, we do need to include every group and every amount of surgeries we considered at least once when executing Algorithm 6.3. Moreover, the latter gives some advise in how many surgeries per session should be scheduled.

7.2 Scenario 2: Surgical duration groups by one standard deviation of mean

In Section 6.4 we have given two different scenarios. The second scenario was mathematical more meaningful. The lower bound of the groups is the mean surgery duration minus the standard deviation, and the upper bound the mean duration plus the standard deviation. For two groups, the boundary is the mean durations. In this section, we will elaborate the sub-scenario's for NCH, URO, KNO, and URK, again.

The R-code we have written to simulate the expected overtime, undertime, and utilization in this case, is given in Appendix D.6.

7.2.1 The combinations of surgery groups

We have calculated the mean, and standard deviation of the surgery durations of each surgical department, see Table 15. Figure 19 gives the groups of departments NCH, URO, KNO, and URK. When we compare these histograms with the one in Figure 17, we see that in this case the first groups are more narrow, and the last group is larger.

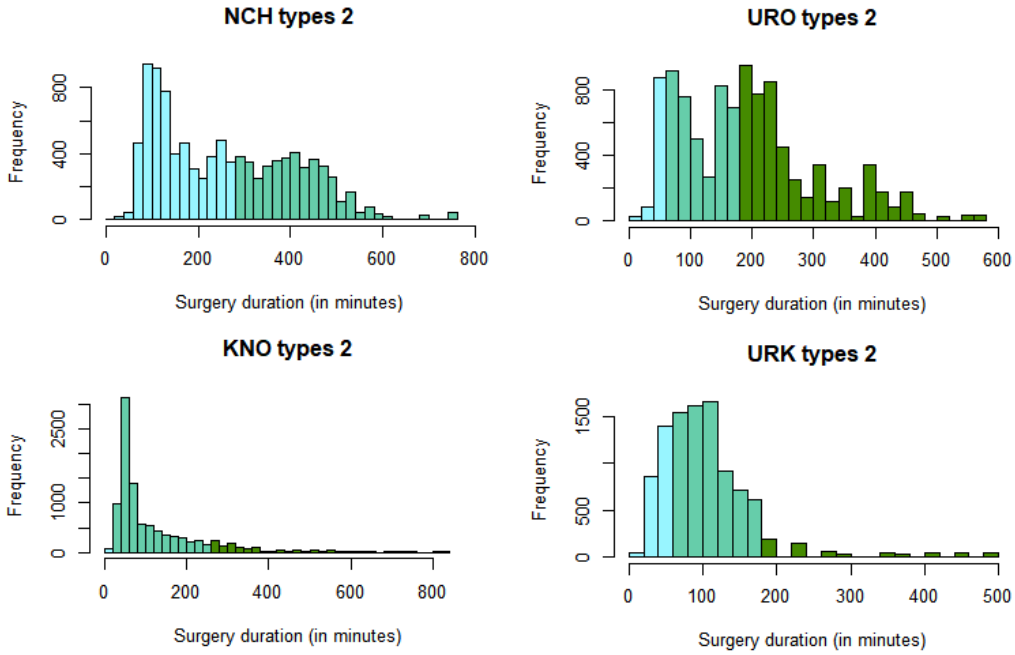


Figure 19: The groups (types 2) for NCH, URO, KNO, and URK

With the same kind of coding as given in Figure 18, but with the boundaries given in Table 15 we determined which groups and combinations of groups are allowed to be scheduled together per session. Again, a group of combination of groups is allowed when the mean duration is larger than 300 and smaller than 600 minutes, or 720 minutes for PCH and KNO.
In Table 15, we see that the mean surgery duration for KNO is 125.54 minutes and the standard deviation is 122.85 minutes, so we have a very low lower boundary, namely 2.69. Therefore, the first group is 'empty', and we can not make an empirical probability distribution for the first group of KNO. So, we have to conclude that this way of determining the surgery duration type is not sufficient for some departments. So, for this scenario we have only looked at the departments NCH, URO, and URK. The next figure, gives the combinations of the surgery durations groups we consider. The complete list of expected overtime, undertime, and utilization of there three departments, can be found in Appendix C.7, with the corresponding conclusions per sub-scenario.

	NCH	URO	URK	
$H_{r,n}$	$\{2\}$	$\{3\}$	$\{3,3\}$	$\{1,1,3,3\}$
	$\{1,1\}$	$\{1,3\}$	$\{1,1,3\}$	$\{2,2,2,3\}$
	$\{1,2\}$	$\{2,2\}$	$\{1,2,3\}$	$\{1,1,1,1,2\}$
		$\{2,3\}$	$\{1,3,3\}$	$\{1,1,1,2,2\}$
		$\{1,1,2\}$	$\{2,2,2\}$	$\{1,1,2,2,2\}$
		$\{1,1,3\}$	$\{1,1,1,3\}$	$\{1,2,2,2,2\}$
		$\{1,2,2\}$	$\{1,1,2,3\}$	$\{2,2,2,2,3\}$
		$\{1,2,2\}$	$\{1,2,2,2\}$	$\{1,1,1,1,3\}$
			$\{1,2,2,3\}$	$\{1,1,1,2,3\}$
			$\{2,2,2,2\}$	$\{1,1,2,2,3\}$

Table 24: combinations type 2

We see that making the surgery duration groups based on the mean surgery duration and standard deviation, yields more allowed combinations of surgeries are allowed, than in the first scenario.

In this case, we have the following utilizations for $S = 470$:

Department	Utilization
NCH	69 %
URO	56 %
URK	57 %

Table 25: Scenario 2: The utilization in case of $S = 470$.

7.2.2 Neurosurgery

Recall, for NCH we have scheduled either one or two surgeries per session. We have divided the surgery durations into two groups, medium long surgeries and long surgeries. The amount of surgeries per session comes with the following probabilities:
The probability of 1 surgery per session: $\mathbb{P}(1) = 0.56$, and
the probability of 2 surgeries per session: $\mathbb{P}(2) = 0.44$.

The combinations of surgery duration groups with a mean duration of more than 300 and less than 600 were:

- 1 long surgery,
- 2 medium long surgeries.
- 1 medium long surgery + 1 long surgery.

Similar to scenario 1, we have calculated the utilization for the current regular session length, which is $b = 69\%$. We rather have a higher utilization, so hopefully the three sub-scenario's give the answer to what the best session length is for NCH. We want a utilization as high as possible, while minimizing the overtime. Note, this utilization is higher than the utilization of NCH in scenario 1 for $S = 470$.

7.2.2.1 Scenario 2.1: Under- and overtime weigh equally

In the case where under- and overtime have gotten the same weight, we get the following session lengths using Eqn. 22:

- For one surgery per session: $S = 390$ minutes.
- For two surgeries per session: $S = 410$ minutes.

Then, the best session length is:

$$S = 0.56 \cdot 390 + 0.44 \cdot 410 \approx 399 \text{ minutes.}$$

Which gives an utilization of approximately 96%.
A high expected utilization might come hand in hand with much overtime. However, for $S = 400$ the expected overtime for one surgery is around 37 minutes, and for two surgeries only around 7.5 minutes. So, the overtime is not that much, but we do have to remember that NCH schedules one surgery much more often per session than two. Therefore, at the end of the year there could still be a lot of overtime.

7.2.2.2 Scenario 2.2: Overtime weighs twice as much as undertime

The more realistic case gives the following session lengths:

- For one surgery per session: $S = 450$ minutes.
- For two surgeries per session: $S = 430$ minutes.

Taking the probabilities of one or two surgeries, we get the session length:

$$S = 0.56 \cdot 450 + 0.44 \cdot 430 \approx 441 \text{ minutes},$$

where $b \approx 72\%$. Which we could determine using Table 26.

7.2.2.3 Scenario 2.3: Minimal utilization of 85% and minimal overtime

In the last sub-scenario, we look at the case where the utilization is 85% for one surgery and for two surgeries per session. The utilization was approximately 85% for the following session lengths:

- For one surgery per session: $S \approx 370$ minutes.
- For two surgeries per session: $S \approx 510$ minutes.

Which gives the session length:

$$S \approx 0.56 \cdot 370 + 0.44 \cdot 510 \approx 432 \text{ minutes},$$

with $b \approx 81\%$. Taking into account the probabilities of one or two surgeries, we do not have the desired utilization of more than 85% anymore. However, the utilization is still quite good. Also, the overtime will be less in this case, than in the first case.

7.2.2.4 Scenario 2 - NCH: Conclusions

In all three cases, we have a session length shorter than 465 minutes. The scenario's 1.1 and 1.3 given the highest utilizations, so we could conclude that the best session length of NCH, when we make the groups bases on the mean surgery duration, is $399 \leq S \leq 432$. The overtime will be highest for $S = 399$, and lowest for $S = 432$.

S	$\mathbb{E}_1[u]$	$\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+2\mathbb{E}_1[o]$	b_1	$\mathbb{E}_2[u]$	$\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+2\mathbb{E}_2[o]$	b_2
290	1.29	114.04	115.33	229.38	133.31%	2.76	46.23	48.99	95.22	182.67%
310	4.28	97.03	101.31	198.34	117.74%	3.86	35.66	39.52	75.18	170.44%
330	8.74	81.49	90.23	171.72	105.04%	5.14	26.57	31.71	58.29	159.67%
350	14.13	66.88	81.01	147.88	96.20%	6.61	19.04	25.65	44.69	150.08%
370	20.94	53.69	74.64	128.33	86.09%	8.32	13.20	21.52	34.73	141.26%
390	29.45	42.20	71.64	113.84	75.86%	10.40	9.03	19.43	28.45	132.82%
410	39.52	32.27	71.79	104.06	67.14%	13.03	6.07	19.09	25.16	124.39%
430	51.37	24.12	75.50	99.62	58.29%	16.39	3.92	20.31	24.23	115.87%
450	64.62	17.37	81.99	99.35	52.00%	20.61	2.34	22.95	25.29	107.56%
470	79.61	12.36	91.96	104.32	44.63%	25.67	1.25	26.92	28.18	100.01%
490	95.99	8.74	104.73	113.48	39.44%	31.76	0.52	32.28	32.79	92.27%
510	113.60	6.35	119.95	126.30	35.48%	38.98	0.13	39.11	39.23	84.62%
530	131.80	4.55	136.35	140.91	34.72%	47.49	0.01	47.50	47.50	76.84%
550	150.81	3.56	154.38	157.94	33.01%	57.31	0.00	57.31	57.31	69.51%
570	170.03	2.78	172.81	175.60	34.23%	68.46	0.00	68.46	68.46	62.59%
590	189.53	2.28	191.81	194.08	35.03%	80.90	0.00	80.90	80.90	56.38%
610	209.12	1.87	210.99	212.85	36.64%	94.57	0.00	94.57	94.57	50.80%

Table 26: expectation in min. utilization percentage NCH - type 2

7.2.3 Urology

Department URO has the following probabilities for the different amount of surgeries per session:
Probability of 1 surgery: $\mathbb{P}(1) = 0.23$.
Probability of 2 surgeries: $\mathbb{P}(2) = 0.61$.
Probability of 3 surgeries: $\mathbb{P}(3) = 0.17$.

Where the following combinations of the three surgery duration groups are allowed:

- 1 long surgery,
- 2 medium long surgeries,
- 1 medium long surgery + 1 long surgery,
- 1 short surgery + 1 long surgery,
- 2 short surgeries + 1 medium long surgery,
- 2 short surgeries + 1 long surgery,
- 1 short surgery + 2 medium long surgeries,
- 3 medium long surgeries.

Again, we calculated the utilization for $S = 470$, which was 56%. Here, we get a lower utilization for URO than we had in scenario 1. We have to conclude that the regular session length of 465 minutes is not sufficient. In other words, URO will or can not use their allocated OR-time properly with their set of surgery durations when they have a session length of 465 minutes.

The results of the simulations are given in Table 27.

7.2.3.1 Scenario 2.1: Under- and overtime weigh equally

When we give the under- and overtime the same weight, their sum is minimal for the following session length when we look at the different amount of surgeries per session separately:

- For one surgery per session: $S = 390$ minutes.
- For two surgeries per session: $S = 410$ minutes.
- For three surgeries per session: $S = 550$ minutes.

Together, these session length give the best session length of :

$$S = 0.23 \cdot 390 + 0.61 \cdot 10 + 0.17 \cdot 550 \approx 433 \text{ minutes.}$$

The corresponding utilization for $S = 433$ is approximately 70%.
So, when we minimize the overtime and undertime, we already get a higher utilization then for $S = 470$. We can assume, that a shorter session length is needed for URO to use their OR-time properly.

7.2.3.2 Scenario 2.2: Overtime weighs twice as much as undertime

In practice, overtime will come with higher cost than undertime. Undertime means that the surgical department leaves some OR-time unused. However, overtime implies most of the time that the departments did not scheduled their surgeries properly. When overtimes occurs, OR-staff needs to work more hours than assigned, this comes with additional wages. Therefore, it is better to make sure as little overtime as possible occurs. This can be accomplished by getting a longer session, which we can determine by giving overtime a higher weight than undertime.
We have determined the session length where for different amount of surgeries the undertime plus twice the overtime is minimal:

- For one surgery per session: $S = 410$ minutes.
- For two surgeries per session: $S = 450$ minutes.
- For three surgeries per session: $S = 590$ minutes.

The best session length is this case is:

$$S = 0.23 \cdot 410 + 0.61 \cdot 450 + 0.17 \cdot 590 \approx 469 \text{ minutes.}$$

Which is 'the same' session length as the current regular session length with the utilization $b \approx 56\%$. Due to the definition of utilization, we obviously get a lower utilization then in the previous case due to minimizing the overtime.

We still have not met the requirement of a utilization of at least 85%. We look into that in the next paragraph.

7.2.3.3 Scenario 2.3: Minimal utilization of 85% and minimal overtime

We have the requirement that a surgical department only performed properly, when they have a utilization higher than 85%. For one, two, and three surgeries per session we get a utilization of 85% for the following session lengths:

- For one surgery: $S \approx 350$ minutes.
- For two surgeries: $S \approx 390$ minutes.
- For three surgeries: $S \approx 510$ minutes.

We take into account the probabilities that we have one, two, or three surgeries per session. This implies the following overall session length:

$$S \approx 0.23 \cdot 350 + 0.61 \cdot 390 + 0.17 \cdot 510 \approx 405 \text{ minutes}$$

The session length $S = 405$ gives URO a utilization: $b \approx 84\%$.

7.2.3.4 Scenario 2 - URO: Conclusions

When we make the surgery duration groups of URO based on the mean surgery duration and the standard deviation of these durations, we see that a session length of 465 minutes is insufficient when we want a utilization of at least 85%. The required utilization is met for $S < 405$ minutes. However, this session length will come with a large amount of overtime.
We have the remark, that in this scenario the groups of short surgeries has a much smaller range in different duration than the groups of long surgeries. We know, that the overtime of a short surgery will be much less than the overtime of a large surgery. It is much harder to reckon with large amounts of overtime, than small amounts. Therefore, it is likely that a longer session length will be chosen to take into account possible overtime, and minimize the extra cost. From the scenario 1.2 is we might conclude that in the current situation, the overtime has given a higher weight than the undertime to make sure the extra cost such as additional wages are minimized. However, the current situation will in this case not ensure a high utilization. So, the session length should supposedly be a bit shorter than 465 minutes.

S	$\mathbb{E}_1[u]$	$\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+$ $\mathbb{E}_1[o]$	$\mathbb{E}_1[u]+$ $2\mathbb{E}_1[o]$	b_1	$\mathbb{E}_2[u]$	$\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+$ $\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+$ $2\mathbb{E}_2[o]$	b_2	$\mathbb{E}_3[u]$	$\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+$ $\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+$ $2\mathbb{E}_3[o]$	b_3
290	0.00	95.00	95.00	189.99	132.76%	3.51	137.85	141.36	279.22	141.16%	0.49	248.86	249.35	498.21	184.61%
310	0.00	75.00	75.00	149.99	124.19%	5.86	120.20	126.06	246.26	128.93%	0.92	229.29	230.22	459.51	172.09%
330	4.33	59.33	63.66	122.99	97.62%	9.23	103.57	112.80	216.38	117.32%	1.60	209.97	211.57	421.55	160.73%
350	10.02	45.01	55.03	100.05	87.30%	13.84	88.19	102.03	190.22	106.08%	2.59	190.96	193.55	384.50	150.37%
370	18.19	33.18	51.37	84.55	73.04%	19.84	74.19	94.03	168.22	95.41%	3.98	172.35	176.32	348.67	140.71%
390	26.75	21.75	48.50	70.24	69.61%	27.29	61.64	88.93	150.57	85.55%	5.85	154.22	160.07	314.29	131.67%
410	39.72	14.71	54.43	69.15	48.44%	36.28	50.62	86.90	137.52	76.28%	8.33	136.70	145.03	281.73	122.98%
430	54.65	9.65	64.30	73.95	40.28%	46.92	41.27	88.19	129.47	67.27%	11.47	119.84	131.32	251.16	114.82%
450	70.87	5.87	76.74	82.61	35.94%	59.02	33.37	92.39	125.76	60.05%	15.42	103.79	119.22	223.01	106.74%
470	89.29	4.29	93.58	97.87	27.82%	72.50	26.84	99.34	126.18	53.76%	20.37	88.74	109.11	197.84	98.49%
490	108.12	3.11	111.23	114.35	28.57%	87.16	21.51	108.67	130.18	48.85%	26.43	74.80	101.22	176.02	90.37%
510	126.94	1.94	128.88	130.82	31.14%	102.82	17.17	119.99	137.15	45.25%	33.55	61.92	95.48	157.40	83.08%
530	146.16	1.16	147.32	148.48	31.70%	119.21	13.56	132.77	146.33	43.08%	41.99	50.36	92.35	142.70	75.25%
550	165.38	0.38	165.76	166.14	34.04%	136.26	10.61	146.87	157.47	41.46%	51.77	40.14	91.91	132.05	67.79%
570	185.00	0.00	185.00	185.00	34.36%	153.79	8.14	161.93	170.07	40.75%	62.92	31.29	94.21	125.50	60.78%
590	205.00	0.00	205.00	205.00	34.75%	171.76	6.10	177.86	183.97	40.32%	75.38	23.75	99.14	122.89	54.49%
610	225.00	0.00	225.00	225.00	36.89%	190.10	4.44	194.54	198.98	40.20%	89.15	17.53	106.68	124.21	48.63%

Table 27: expectation in min. utilization percentage URO - type 2

7.2.4 Pediatric Urology (URK)

Surgical department URK mostly schedules two, three, four, or five surgeries per session. In Section 6.4, we have determined that these amounts have the following probabilities:
Probability of 2 surgeries per session: $\mathbb{P}(2) = 0.14$.
Probability of 3 surgeries per session: $\mathbb{P}(3) = 0.43$.
Probability of 4 surgeries per session: $\mathbb{P}(4) = 0.25$.
Probability of 5 surgeries per session: $\mathbb{P}(5) = 0.18$.

After we have made the three groups, we determined which combinations of these groups are allowed. We get much more combinations then in scenario 1:

- 2 long surgeries,
- 1 short + 1 medium long + 1 long surgery,
- 2 short surgeries + 1 long surgery,
- 1 short surgery + 2 long surgeries,
- 3 medium long surgeries,
- 3 short surgeries + 1 long surgery,
- 2 short surgeries + 1 medium long + 1 long surgery,
- 1 short surgery + 2 medium long surgeries,
- 1 short surgery + 2 medium long surgeries + 1 long surgery,
- 4 medium long surgeries,
- 2 short + 2 long surgeries,
- 3 medium long surgeries + 1 long surgery,
- 4 short surgeries + 1 medium long surgery,
- 4 short surgeries + 1 long surgery,
- 3 short surgeries + 2 medium long surgeries,
- 2 short surgeries + 3 medium long surgeries,
- 1 short surgery + 4 medium long surgeries,
- 5 medium long surgeries,
- 2 short + 2 medium long surgeries + 1 long surgery,
- 3 short surgeries + 1 medium long + 1 long surgery.

The utilization with this groups will for the regular session length $S = 470$ minutes be 57%. Just like URO, URK has a lower utilization in scenario 2 than in scenario 1.

7.2.4.1 Scenario 2.1: Under- and overtime weigh equally

When we assume that undertime and overtime are equally as bad, i.e. they have the same amount of extra cost, the following sessions are determined using Eqn. 22:

- For two surgeries per session:$S = 430$ minutes.
- For three surgeries per session: $S = 350$ minutes.
- For four surgeries per session: $S = 490$ minutes.
- For five surgeries per session: $S = 550$ minutes.

The best session length follows then we use the chances on the different amount of surgeries on the session:

$$S = 0.14 \cdot 420 + 0.43 \cdot 330 + 0.25 \cdot 450 + 0.18 \cdot 550 \approx 412 \text{ minutes.}$$

$S = 412$ gives a utilization: $b \approx 73\%$. This is a good utilization, much higher than when URK would have $S = 465$, but we still want the utilization to be higher.

The next sub-scenario will give a lower utilization, because we need a longer session length to reckon with the overtime with a higher weight than the undertime. Recall, this is due to the fact of how utilization is defined, see Eqn. 9.

7.2.4.2 Scenario 2.2: Overtime weighs twice as much as undertime

We have given overtime a higher weight than undertime, because overtime will give higher extra cost. Therefore, we want to make sure that less overtime will occur, which can be accomplished by assigning URK a longer session length. For the different amount of surgeries URK schedules per session follows from Eqn. 23

- For two surgeries per session: $S = 490$ minutes
- For three surgeries per session: $S = 350$ minutes
- For four surgeries per session: $S = 490$ minutes
- For five surgeries per session: $S = 590$ minutes

The best session length is now:

$$S = 0.14 \cdot 490 + 0.43 \cdot 350 + 0.25 \cdot 490 + 0.18 \cdot 590 \approx 448 \text{ minutes,}$$

with $b \approx 60\%$. The best session length in this case is still lower than the current regular session length of 465 minutes. We can conclude from this that that session length is neither a sufficient length for a high utilization, nor for a minimized amount of overtime (when we also still take into account the undertime).

7.2.4.3 Scenario 2.3: Minimal utilization of 85% and minimal overtime

In the previous two cases we did not get a utilization higher than 85%, which is one of the objectives. A higher utilization occurs when there is more overtime, so we will get a short session length. However, the discussion whether high utilization or low overtime is more important should be started, because we know from the literature we have read (Section 4.1), that one long session is better than multiple short ones.
For the following session lengths, we get a utilization of approximately 85% for different amount of surgeries that can be scheduled per session:

- For two surgeries per session: $S \approx 410$ minutes.
- For three surgeries per session: $S \approx 310$ minutes.
- For four surgeries per session: $S \approx 420$ minutes.
- For five surgeries per session: $S \approx 610$ minutes .

These session length together, give the best session length:

$$S \approx 0.14 \cdot 410 + 0.43 \cdot 310 + 0.25 \cdot 420 + 0.18 \cdot 610 \approx 406 \text{ minutes.}$$

From the results in Table 28, we know that for $S = 406$ $b \approx 76\%$. This session length is almost one hour shorter than the regular session length that exists currently.

7.2.4.4 Scenario 2 - URK: Conclusions

From the session lengths determined in the three sub-scenarios we have to conclude that URK needs a session length shorter than the current regular session length. The session length $S = 470$ does implies low expected overtime, but gives also a very low utilization. So one should question what is more important. Low amount of overtime, might mean a high amount of undertime. Overtimes has higher extra costs such as additional wages for the staff, but unused ORs will also induces some costs one would want to avoid.

We have to remark from Table 28, that is is probably not sufficient to give departments URK only one type of session length. We will discuss this further in Section 9.

S	$\mathbb{E}_2[u]$	$\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+\mathbb{E}_2[o]$	$\mathbb{E}_2[u]+2\mathbb{E}_2[o]$	b_2	$\mathbb{E}_3[u]$	$\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+\mathbb{E}_3[o]$	$\mathbb{E}_3[u]+2\mathbb{E}_3[o]$	b_3
290	0.00	24.91	24.91	49.82	166.13%	7.81	47.13	54.94	102.07	99.25%
310	0.00	22.31	22.31	44.63	155.41%	14.10	33.42	47.52	80.93	83.89%
330	0.00	19.72	19.72	39.43	145.99%	23.12	22.44	45.56	67.99	68.71%
350	0.00	17.12	17.12	34.24	137.65%	34.82	14.14	48.96	63.10	55.47%
370	0.10	14.62	14.72	29.34	126.8%	49.06	8.38	57.44	65.82	44.33%
390	0.82	12.75	13.57	26.32	98.88%	65.42	4.73	70.15	74.89	36.2%
410	1.81	11.13	12.94	24.07	86.52%	83.28	2.59	85.87	88.47	31.65%
430	3.03	9.75	12.78	22.53	75.90%	102.17	1.49	103.66	105.14	29.65%
450	4.47	8.60	13.07	21.67	66.70%	121.64	0.96	122.60	123.56	29.89%
470	5.96	7.49	13.45	20.94	64.75%	141.37	0.69	142.06	142.75	31.58%
490	7.65	6.58	14.23	20.80	57.45%	161.22	0.54	161.76	162.30	33.75%
510	9.38	5.72	15.10	20.82	55.92%	181.13	0.44	181.57	182.02	36.08%
530	11.14	4.87	16.01	20.89	55.76%	201.05	0.36	201.41	201.77	38.41%
550	12.97	4.11	17.07	21.18	53.48%	220.98	0.30	221.27	221.57	40.57%
570	14.86	3.40	18.27	21.67	51.69%	240.92	0.23	241.15	241.39	42.61%
590	16.86	2.80	19.66	22.46	48.86%	260.86	0.18	261.05	261.23	44.51%
610	18.90	2.25	21.15	23.40	47.93%	280.82	0.14	280.96	281.09	46.28%

S	$\mathbb{E}_4[u]$	$\mathbb{E}_4[o]$	$\mathbb{E}_4[u]+\mathbb{E}_4[o]$	$\mathbb{E}_4[u]+2\mathbb{E}_4[o]$	b_4	$\mathbb{E}_5[u]$	$\mathbb{E}_5[o]$	$\mathbb{E}_5[u]+\mathbb{E}_5[o]$	$\mathbb{E}_5[u]+2\mathbb{E}_5[o]$	b_5
290	0.11	166.02	166.12	332.14	156.84%	0.00	261.45	261.45	522.90	190.15%
310	0.36	146.27	146.64	292.91	146.02%	0.01	241.46	241.46	482.92	177.87%
330	0.98	126.90	127.88	254.78	135.64%	0.01	221.46	221.48	442.94	167.07%
350	2.24	108.15	110.38	218.53	125.28%	0.04	201.49	201.53	403.02	157.45%
370	4.40	90.31	94.71	185.01	114.79%	0.12	181.57	181.69	363.26	148.71%
390	7.90	73.81	81.72	155.53	103.43%	0.30	161.75	162.06	323.81	140.64%
410	13.11	59.02	72.13	131.15	91.56%	0.68	142.13	142.81	284.94	132.95%
430	20.31	46.22	66.53	112.75	79.47%	1.38	122.83	124.22	247.05	125.36%
450	29.55	35.46	65.01	100.47	68.24%	2.68	104.13	106.81	210.94	117.25%
470	40.81	26.72	67.53	94.25	58.06%	4.92	86.37	91.29	177.65	108.24%
490	54.04	19.95	73.99	93.94	48.95%	8.44	69.89	78.32	148.21	98.39%
510	68.97	14.88	83.84	98.72	41.81%	13.51	54.96	68.48	123.44	88.04%
530	85.23	11.14	96.38	107.52	36.85%	20.46	41.91	62.37	104.28	77.04%
550	102.67	8.58	111.25	119.83	33.05%	29.39	30.84	60.23	91.07	66.31%
570	120.81	6.72	127.53	134.24	31.68%	40.34	21.79	62.12	83.91	56.16%
590	139.45	5.36	144.80	150.16	31.35%	53.31	14.76	68.08	82.84	46.65%
610	158.44	4.35	162.78	167.13	31.73%	68.04	9.49	77.52	87.01	39.10%

Table 28: expectation in min, utilization percentage URK - type 2

7.3 Discussion on the regular session length

The first research question concerns the best session length for each surgical department, when we only allow a fixed session length per surgical department. In the current session distribution departments get a regular session length of 465 minutes, and some departments get some additional longer session lengths. In 2017, CCH has been allocated a session length of 585 minutes almost every day, NCH every two weeks one of 570 minutes, and KNO and PCH have been allocated an occasional session of 705 minutes sometimes. Now suppose we want departments to be allocated one fixed session length to make the schedule more transparent. In Section 7.1 and 7.2, we have simulated the expected overtime, undertime, and utilization for each surgical department for different session lengths and different amounts of surgeries per session. The simulation results provide the basis for determining the best session length for each surgical department, and are given in Appendix C.4. The best session length of each surgical department will help us to conclude whether variable session lengths contribute to a better OR utilization. Here, we have rounded the session lengths to either to a whole hour, 15 minutes over or before the hour, or half past the hour (as is currently done).

We do not have the same experience as the staff working at the OR complex and at the planning bureaus, therefore we have looked into different scenarios to determine the best session length per surgical department. We will not be able to give a specific amount of minutes a surgical departments should have in their own regular session length, because the discussion per high utilization versus a low amount of overtime is yet to be conducted. We want a utilization of more than 85% and minimal overtime, but higher utilization implies more overtime. However, sometimes a bit more expected overtime will give a much higher utilization. There are no rules written down yet over to what extend overtime is allowed to get a higher utilization. This is all done by experience of the hospital staff, we do not have. Although we can not give an exact length of the best session for each departments, we can give recommendations on whether a department should get a longer or shorter session length, or that the assigned session length in the session distribution of 2017 is satisfactory.

The list below summarizes our recommendations. Note that scenario 2 has not been implemented for each department, because in those cases it was not possible to make an empirical probability distributions for each surgery duration group. We have only implemented scenario 2 for NCH, URO, and URK. Therefore, only for these three departments we can include the observations of both scenarios in our recommendations.

- **CCH:**
  In 2017 CCH has been allocated a session of 585 minutes almost every day. From Table 23 we know that CCH indeed needs a longer session length than 465 minutes. However, 585 is not sufficient because is will yield a low utilization. The best session length for CCH will be between 500 and 555 minutes. A difference of 30 minutes with the current situation might not be worth the effort, but approximately 80 minutes extra will be satisfactory. Also, for $S = 470$ the utilization of CCH is 100% (see Table 22) which implies that for $S = 470$ the session is to short and we will experience overtime.

- **LON:**
  In each case in scenario 1, this department has a session length with a utilization that could be good enough. However, we do see that the session length in each case is smaller than 465 minutes. Table 22 shows that session length $S = 470$ will imply $b \approx 59\%$, which is way too low to be acceptable. Therefore, it would be best to decrease the regular session by with at most 60 minutes for LON.

- **ORT:**
  For surgical department ORT we have to conclude that the current session length is sufficient.

- **HLK:**
  For HLK the session length of 465 is insufficient. We have seen in Table 23, that a session length of 325 minutes gives a utilization of 100%, but a utilization of 100% will in our situation imply overtime (because we look at only one session length, and the expected utilization). Therefore, a longer session length might be preferable. For $S = 358$ minutes, we almost have a utilization of 85%. We advise to assign HLK sessions of length between 330 and 360 minutes.

- **NCH:**
  NCH needs a session length of 30 to 60 minutes shorter than the regular session length in the session distribution of 2017, in case of scenario 1.
  In case of scenario 2, we would again advise to decrease the session length of 465 by circa 30 to 60 minutes.

- **GYN:**
  In Table 22 we have seen that $S = 470$ is a very bad session length for GYN. Best is to decrease the 465 minutes session by 60 to 100 minutes.

- **URO:**
  Decreasing the session length circa 30 minutes will give a expected utilization of 86%, in scenario 1. However, when minimizing the overtime by giving it a higher weight than undertime, we would want $S \approx 510$. Therefore, it might not be optimal to decrease the session length of 465 minutes for URO based on scenario 1. In case of scenario 2, we have to conclude that the $S = 470$ is not sufficient, because it gives a utilization of only 56%. Based on scenario 2, the session length of URO should be between 405 and 435 minutes.

- **Gender:**
  From the results of scenario 1.2, we can conclude that Gender needs a session length shorter than 445 minutes to get a utilization higher than 50%. Therefore, a session length of $405 \leq S \leq 375$ is preferred (rounded to useful hours).

- **MKA:**
  For department MKA it is best to conclude that the current session length is satisfactory.

- **PCH:**
  PCH has gotten a extra long session of 705 minutes every four weeks in the session distribution of 2017. The other sessions of PCH in the session distribution of 2017 are of regular length. In this research question, we decided to allow only one session length per surgical department. Therefore, the best session length of PCH will be between the 465 and 705 minutes. From Table 23 we can conclude that PCH indeed needs a higher session length than 465, when we allow only one session length.

- **KNO:**
  Next to PCH, KNO has been assigned a extra long session of 705 minutes as well. In the conclusions of scenario 1 we have seen that is session length with the highest utilization is circa one hour shorter than 465 minutes. However, when we gave the overtime a weight twice as much as the undertime, we have gotten a session length of almost 570 minutes with a low utilization. This can be explained by the large top in the histogram of the surgery durations of KNO. We have to conclude that we can not give a proper answer to the first research question for KNO.

- **OOG:**
  In our model, a session length of 465 minutes is insufficient for department OOG, see table 22. With Table 23, we can conclude that OOG needs a session length between circa 60 and 105 minutes shorter than the current regular session length they have been assigned in 2017. However, in the list with results of OOG, we see that the best session length of one of two surgeries per session is very short. We have mentioned in 4.1 that longer sessions are better than shorter ones. Therefore, is would be much better to schedule only for or five surgeries per session. Then, the best session length is approximately 465 minutes.

- **KCH:**
  Table 23 shows that a slightly shorter session length gives a proper utilization. When we take into account that overtime implies more costs than undertime, we want to have a session length that is circa 45 minutes longer. However, to make sure to get as little overtime as possible, we have to be compromise with a low utilization. Which we rather do not have. Therefore, is might be best not to change the regular session length of KCH.

- **URK:**
  In scenario 1, a session length of 465 minutes is sufficient. The highest utilization has been achieved for $S = 426$, but a high the utilization implies a high expected amount of overtime.
  However, in scenario 2 $S = 465$ is insufficient. In this case, we rather want to decrease the session length by 60 minutes.

7.4 Answer to research question 1

We summarize our observations and recommendations from the previous section to answer the first research question:

"*What is the optimal session length per surgical department, when they can have only one length?*"

- In 2017, the departments CCH, ORT, KCH, MKA, and PCH have been assigned session lengths that are close to satisfactory. The best session length for ORT, KCH, and MKA is 465 minutes. CCH should have a session length between 500 and 555 minutes, and PCH should probably have a session length within this range as well.
- HLK should get a session length between 330 and 360 minutes.
- Surgical departments LON, NCH, GYN, URO, and Gender should all have a session length shorter than the current regular session length of 465 minutes. URO should get a session length up to 30 minutes shorter, NCH between 30 and 60 minutes shorter, and LON should get a session length up to 60 minutes shorter. Department GYN should have a session length between 360 and 405 minutes, and Gender between 300 and 375 minutes.
- Also for OOG a shorter session length of 360 to 405 minutes should be assigned according to our model. However, our model shows that two or three surgeries per session is not desirable. When only 4 or 5 surgeries are scheduled, 465 minutes would be sufficient.
- For the last two departments, KNO and URK, we have not been able to give a conclusive answer on the best session length.

We will make some remarks. The conclusion is based on simulations of the empirical probability distributions as explained in Section 6, the results in Sections 7.1 and 7.2, and on the assumptions that have been made in section 6.3. Therefore, the overtime and undertime could be higher than actually would happen in practice. Also, the best session lengths could deviate from the best session length based on the precise combinations of surgeries that occur. We have made the assumption that we are allowed to divide the surgery durations of a departments into groups, this might be in practice not possible or hard to accomplish. Further, when we are indeed allowed to make groups, we might compose the groups for each department differently. Which will lead to alternative potential combinations.

Now that we have a insight on the best session length per department, we interested in how a blueprint based on these session lengths would look like. We will elaborate this in the next chapter.

8 Results RQ2: A new blueprint

The second research question is:

"*What is the optimal blueprint design concerning the answer of research question 1?*".

Based on our recommendations on the best session length per surgical department, we next suggest a new blueprint based on our findings.

Maximally 15 operation rooms are available each day for the elective program, and there are 252 elective days. So in total we have maximal $15 \cdot 252 = 3780$ elective OR-days. Recall, in holiday periods less staff is available, so less than 15 ORs per day can be used in those periods. This is one of the reasons the total amount of sessions allocated and used in Table 3 is lower than 3780. Recall we also have excluded some data, see Section 3. We also want to take into account the results of Section 3.4.2.

We have used the allocated OR-time in 2017 to calculate the number of sessions needed per department. The table below contains the data from Table 4, and the number of sessions needed based on the answer of research question 1. For most departments, we only gave a range for the session lengths. Therefore, we have calculated the amount of sessions for the lowest and the highest advised session length.
Remark, in Section 7.4 we have concluded that for KNO and URK we could not determine the best session length based on the scenarios we ran. Therefore, we have chosen a broad range for the session length of KNO, and for URK we have chosen to use the result of scenario 1 as an upper bound and the result of scenario 2 as a lower bound, see Section 7.3.

				Amount of sessions	
C	A	S_A	Best session length	Lower bound	Higher bound
NCH	146,625	311	405-435	362	337
CCH	220,290	469	500-555	441	397
HLK	312,195	664	330-360	946	867
LON	50,370	106	405-465	124	108
Gender	47,355	100	300-375	158	126
URO	72,330	154	435-465	116	156
GYN	117,210	248	360-405	326	289
ORT	67,710	144	465	146	
PCH	66,465	142	500-555	133	120
MKA	52,545	112	465	113	
KNO	202,620	431	405-570	500	356
OOG	106,020	225	360-405	295	262
URK	42,060	89	405-465	104	90
KCH	41,190	88	465	89	
TOTAL		3,283		3,598	3,270

Table 29: Amount of sessions per surgical department for the new session lengths

With the 252 elective OR-days, the surgical departments NCH, CCH, GYN, KNO, and OOG need multiple days with two ORs assigned, and HLK needs two or three ORs every day. In 2017, CCH was allocated two ORs almost every day, NCH two ORs twice a week, and KNO has 10 ORs in total per week. Also, HLK has four days with three ORs available most weeks, and one day two ORs. Therefore, for these four departments their session length could vary, but the amount of ORs per day will not. However, GYN and OOG both have one allocated OR available each elective day in 2017.
We have concluded that a session length of 465 minutes is best for ORT, MKA, and KCH. These three only have regular length sessions in 2017, so for those departments nothing will change by implementing the best session length. We have advised for the other departments that a shorter session length is desirable in order to maximize utilization. Therefore, they should be allocated more sessions than in 2017.

9 Results RQ3: Multiple session lengths

This research was aimed at determining the individual best session length for each department. It turned out not to be a good choice to assign only one single session length, minimal two different lengths would be better. We have decided to assign departments not more than two different session lengths, to make sure that no confusion can arise on what session length is assigned on what day.
To be able to determine what the two best session lengths are for a surgical department, it is not satisfactory to only base the session length on the probability of a surgery duration and of the surgery groups. We need to of the predictability of a surgery duration. The surgeon, the age, gender, or weight of a patient, can all affect predictability. It is beyond the scope of this research to analyze this. However, we are able to give some insight on which departments most likely will need to get multiple session lengths to get a high utilization with minimal overtime.
One does have to note that some of our assumptions influence the outcome on this research question. First, we only took into account the amount of surgeries per session that had a frequency of at least 10% in 2015. We also excluded some surgery combinations that normally would have been planned together, and included some surgeries that in practice never would have been planned together.

For example, department KNO has a very high frequency of surgery durations between 20 and 80 minutes (see the figure below). Our model than yields a relatively small best surgery duration. In Section 7.3 we have concluded that the best session length based on our model would be approximately 405 minutes. However, with $S = 405$ the expected overtime will be large when we want to schedule a surgery that belongs to the medium long or long surgery duration group (see Figure 21). In 2017, KNO has been assigned mostly sessions of the regular length of 465 minutes, but also a session length of 705 minutes per weeks. The existing situation already took into account that it is best to assign KNO two different session lengths. However, based on our situations, we would prefer to assign KNO a session length of 405 minutes and one of 570 minutes. We will elaborate this in the following example.

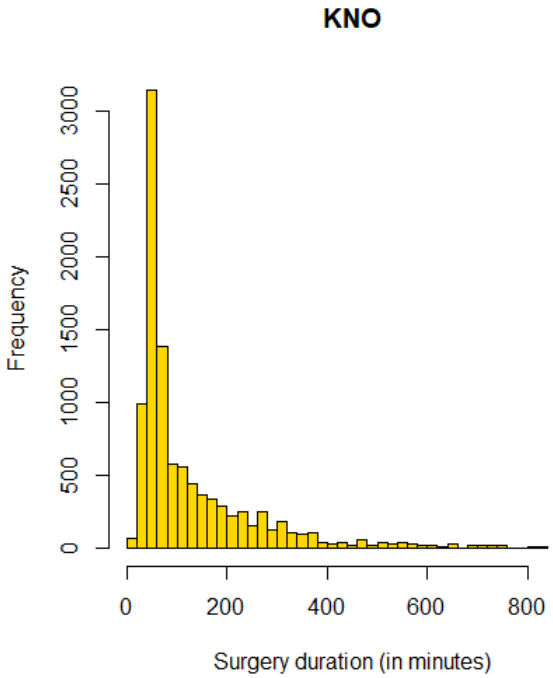


Figure 20: Histogram on the (statistical significant) surgery duration of KNO

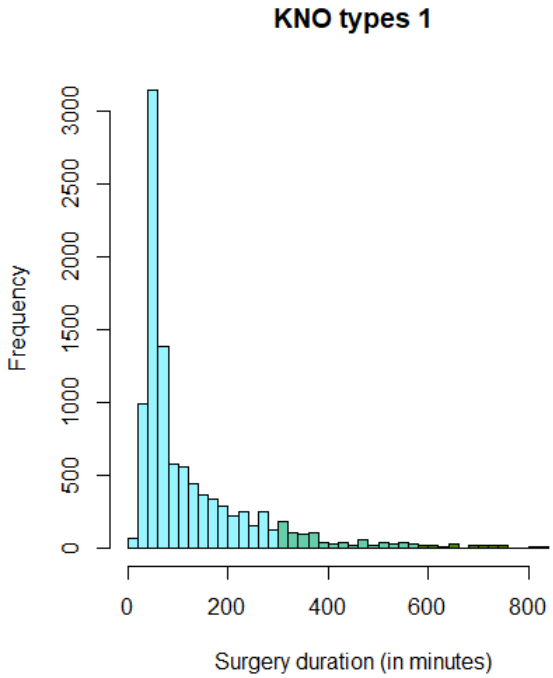


Figure 21: The groups of KNO in scenario 1

Example 9.1.
Suppose we assign KNO two session lengths: $S_1 = 405$, and $S_2 = 570$, based on minimizing overtime and the utilization around 85%. Further, KNO schedules either 1,2,3,4, or 5 surgeries per session with: $\mathbb{P}(1) = 0.18$, $\mathbb{P}(2) = 0.29$, $\mathbb{P}(3) = 0.19$, $\mathbb{P}(4) = 0.20$, and $\mathbb{P}(5) = 0.14$. Then, we have S_1 with probability 0.18+0.29+0.19+0.20=0.86, and S_2 with probability 0.14.
Now suppose we decide that 1, 2, 3, or 4 surgeries are schedules on S_1, and 5 surgeries on S_2. Then, we have to re-normalize the probabilities of 1, 2, 3, or 4 surgeries per session:
$\mathbb{P}_{S_1}(1) = \frac{0.18}{0.18+0.29+0.19+0.20} = 0.21$,
$\mathbb{P}_{S_1}(2) = 0.34$,
$\mathbb{P}_{S_1}(3) = 0.22$,
$\mathbb{P}_{S_1}(4) = 0.23$,
and, $\mathbb{P}_{S_2}(5) = 1$.

Using Tables 20 and 21, we can determine the utilization for S_1 and S_2:
$b_{S_1} = 0.21 \cdot 66.5 + 0.34 \cdot 87.0 + 0.22 \cdot 64.0 + 0.23 \cdot 86.8 \approx 87\%$.
$b_{S_2} = 62.0\%$.
So the overall expected utilization will be: $b = 0.86 \cdot 78 + 0.14 \cdot 62 \approx 76\%$.

When we use the lists of expected overtime, undertime and utilization (Appendix C.5 and C.7.), we can see that for some of the surgical departments the best session length differs a lot between the different amount of surgeries per session. The way we make the groups is affected by whether multiple session lengths are needed or not. For example, in scenario 1 a session length of 465 minutes might be sufficient for URO, but in the case of scenario 2 it would be better to give URO an additional longer session, see Tables 18 and 27.
The lists of MKA and ORT in scenario 1 show that for four surgeries per session all three sub-scenarios assign a far longer session length than when scheduling less surgeries. Therefore we could conclude that these two departments should get a longer session of at least 585 minutes. However, it might be that this occurred due to our modeling assumptions. If we do not want to assign a long session length to these two departments, they should no longer schedule four surgeries per session. Further, from the results given in Appendix C.5, we can conclude that it was necessary to give department PCH two different session lengths, one for 1,2, or 3 surgeries per session, and one for 4 surgeries.

Example 9.2.
Suppose PCH schedules either 1, 2, or 3 surgeries per session of $S_1 = 465$, and 4 surgeries on $S_2 = 705$. The number of surgeries per session have probabilities: $\mathbb{P}(1) = 0.30$, $\mathbb{P}(2) = 0.18$, $\mathbb{P}(3) = 0.35$, and $\mathbb{P}(4) = 0.17$. We assign S_1 with probability 0.30+0.18+0.35=0.83, and S_2 with probability 0.17. We have to re-normalize these probabilities. We get:
$\mathbb{P}_{S_1}(1) = \frac{0.30}{0.30+0.18+0.35} = 0.36$,
$\mathbb{P}_{S_1}(2) = 0.22$,
$\mathbb{P}_{S_1}(3) = 0.42$,
$\mathbb{P}_{S_2}(4) = 1$.

With the list of expected overtime, undertime, and utilization of PCH in Appendix C.4, we determine the expected utilization for those two session lengths:
$b_{S_1} = 0.36 \cdot 89.60 + 0.22 \cdot 105.4 + 0.42 \cdot 53.94 \approx 76\%$,
$b_{S_2} = 54.91\%$.
So, $b = 0.83 \cdot 76 + 0.17 \cdot 54.91 = 72\%$.

As given in Section 7.3, it depends on whether one and two surgeries per session will continue to be allowed for department OOG whether they need two different session lengths. When we do allow it, OOG should get a session length not only of 465 minutes, but also one much shorter. We have noticed the same for departments LON, when looking at one surgery per session. Department HLK should also get a session length of 465 (preferable when 3 surgeries are scheduled per session), and a shorter one of approximately 370 minutes.

Remarkable is that from our results in case of scenarios 1 and 2 it can be concluded that one session length for NCH is sufficient, and that a session length of 465 is less is optimal, although NCH also has some sessions of 570 minutes in the current situation. The department is also requesting more longer sessions for quite some time now. Recall, for NCH we have only looked at one of two surgeries per session. IF we would have allowed three surgeries, we might have gotten that some longer sessions are indeed necessary. Another remarkable observation is that CCH had a session of 585 minutes and a session of 465 minutes every day in 2017. Our results suggest however, that a session of 465 is never satisfactory. Further, for Gender and GYN it is sufficient to assign one session length.

The answer on research question 3:

"*What is the optimal combination of session lengths per surgical department, when we allow different lengths?*"

- Departments CCH, NCH, Gender, and GYN do only need one session length. The one given in Section 7.4.
- The session lengths assigned to PCH in the blueprint of 2017, $S_1 = 465$ and $S_2 = 705$ minutes, should remain the same. The sessions of PCH should be S_1 0.82 of the time, and S_2 otherwise.
- KNO needs two session lengths of circa $S_1 = 405$ minutes, and $S_2 = 585$ minutes respectively. We advise to assign 0.86 of all sessions of KNO length S_1, and the other sessions length S_2.
- For URO it really depends on how the surgeries are divided into groups, whether one session length is enough to get optimal results.
- Due to the assumptions made in this model, we can not give a conclusive answer whether it is indeed better to assign ORT and MKA a longer session of at least 585 minutes.
- The departments OOG, HLK, and LON should get a regular session length of around 465 minutes, and a shorter one.

When the new blueprint is being made, one should use the session lengths determined in Sections 7.4 and 9. It is preferable to change not that much about the session allocation, therefore also the data-analysis in Section 3.4.2, and Appendix C.2.

10 Conclusion

Now that we have answered the three research questions we can answer the research objective. Our research objective is:

"*Do variable session lengths contribute to a better OR utilization?*"

The first thing that makes answering this question difficult, is the fact that the definition of utilization in the model to determine the session distributions of 2017 has not been defined properly. That definition does not include the turnover times, therefore the used OR-time is less than it has actually been. Hence, we can only compare the utilization for the new determined best session lengths (research questions 1 and 3) with the utilization for $S = 470$ in our model. We have only looked at the utilization of one session. We have not done a simulation of what a whole year could have looked like, in which we would not have infinite pool of surgeries to choose from. This simulation would have had an influence on the utilization at the end of the year.

We have concluded that for most departments a shorter session length than 465 minutes suffices. Therefore, overtime and undertime can easily be explained in the current situation. Undertime and overtime are both not desirable. When the session is shortened in time, less undertime will occur per session, and an additional session becomes available. Recall Examples 1.7, and 5.4.

Given the answers of research questions 1 and 2, we can give the following answer to the research objective:

The goal of the OR departments is to improve the utilization, while the overtime should be as low as possible. We have seen that for most departments some gain can be obtained by changing their regular session length, and assigning some departments an additional session length as well. Comparing the session lengths assigned to each department in 2017 and the session lengths we have determined, some changes have to be made. Fortunately, not that much change is needed. Another positive part of the new determined session lengths is the fact that it is still possible to assign most departments the same optimized session length. Hence, there will be variable session lengths, but not that many. Therefore, changing to the new best session lengths would be realistic.

11 Discussion and future work

In this research multiple assumptions were made in order to answer the research questions and objective function. These assumptions will influence the outcome of the research. Also, we have looked into two different scenarios to make the problem a bit more scalable. However, we did not do any research on what the best scenarios and sub-scenarios were to execute. Therefore it is preferable to continue the research under different circumstances (scenarios) with different and/or less assumptions. Also, more knowledge on the surgeries that are likely to be scheduled together will give a more accurate solution.
The expected overtime, undertime, and utilization have been determined using the probabilities of different numbers of surgeries per session, and the distribution of surgery duration groups combinations. In the future, research should be done on the predictability of a surgery duration. We have simulated 'all' possible outcomes of surgery durations per session to determine the best session lengths to use in the blueprint. Moreover, when making a surgery schedule one would like to know the durations of the surgeries to be scheduled as precise as possible.
An important follow-up step on this research is to do simulations on an entire year. In other words, finish research question 2 and do simulations on all sessions a surgical department has been assigned in order to determine the utilization per year. Also, we have only taken into account the amount of surgeries per session based on the realized data of 2015. This research should be expanded by allowing less or more surgeries per session than currently have been done. By doing that, we will even get a better answer on the best session length. We have seen some departments that should get shorter session lengths (to minimize undertime) than they currently have, but with a longer session length they might do another surgery per session.

The current regular session length is based on a typical working day length of the staff. When new session lengths are implemented, we need to think about the consequences on the working hours of the staff. Next to that, a discussion has to be started on the importance of high utilization versus low overtime. With the definition of utilization we have advised in Section 4.3, a very high utilization will be obtained when allowing more overtime. This is not preferable due to extra cost, but there are no regulations on how much overtime is allowed to get a better utilization. A couple of minutes overtime might be much better than many minutes of undertime.

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